Last Book: Geometric Representation

May 23-9, 2023

Lesson 1 : Laws of Exponents and Simplifying Exponential Expressions

Recall: Rewriting a string of Addition:

$2 + 2 + 2 = 3 \times 2$

$2 + 2 + 2 + 2 = 4 \times 2$

$\underbrace{2 + 2 + \cdots + 2}_{10 \text{ times}} = 10 \times 2$

$\underbrace{2 + 2 + \cdots + 2}_{n \text{ times}} = n \times 2$

Note: Rewriting a string of multiplication

$2 \times 2 = 2^2$ (this reads, two squared)

$2 \times 2 \times 2 = 2^3$ (this reads, two cubed)

$2 \times 2 \times 2 \times 2 = 2^4 = (16)$ (two to the four)

$\underbrace{2 \times 2 \times \cdots \times 2}_{10 \text{ times}} = 2^{10}$

$\underbrace{2 \times 2 \times \cdots \times 2}_{n \text{ times}} = 2^n$ ← the exponent (the power)

Terminology: the base of the exponential number

Calculators:

buttons:

x^2

x^3

y^x

$\wedge$

Ex. Rewrite in Exponential Notation

$13 \times 13 \times 13 \times 13 = 13^4$ $(= 28561)$ base $= -3$

$(-3 \times -3)(-3 \times -3) = (-3)^4$ $(= 81)$

$(-3 \times 3) \times 3 \times 3 = -3^4 = -1 \cdot 3^4$ $(= -81)$

coefficient ↑ ↑ base $= 3$

$\frac{4 \times 4}{5 \times 5 \times 5} = \frac{4^2}{5^3}$

$\frac{-2 \times 2}{-3 \times -3 \times -3} = \frac{-1 \cdot 2^2}{(-3)^3}$

$m \times m \times m = m^3$

$\cdot\cdot \times \cdot\cdot = \cdot\cdot^2$

$x \cdot x \cdot x \cdot y \cdot y = x^3 \cdot y^2$

$(2+y)(2+y) = (2+y)^2 = (2+y)\cdot(2+y)$

$\neq 2^2 + y^2$

$= 2^2 + 2y + 2y + y^2$

$= 2^2 + 2(2y) + y^2$

$= 4 + 4y + y^2$

$(x-1)(x-1)(x-1) = (x-1)^3$

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3$

$\frac{1}{2}$ to the three

$\frac{1}{2}$ cubed

ex : Rewrite / Expand

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \; (= 729)$

base = −2

$(-2)^3 = (-2) \times (-2) \times (-2) \; (= -8)$

base = 2

$-2^3 = -2 \times 2 \times 2 \quad (= -8)$

$= -1 \cdot 2^3$

$\frac{7^4}{10^2} = \frac{(7 \times 7 \times 7 \times 7)}{10 \times 10}$

$\frac{(-6)^2}{-5^4} = \frac{-6 \times -6}{-5 \times 5 \times 5 \times 5}$

$\left(\frac{2}{3}\right)^2 = \frac{2}{3} \times \frac{2}{3}$

$q^4 = q \times q \times q \times q$

base = x

$-x^2 = -1 \cdot x^2 = -1 \cdot x \cdot x$

base = −y

$(-y)^3 = -y \cdot -y \cdot -y$

$(x+y)^2 = (x+y) \cdot (x+y)$

$(-x+y)^3 = (-x+y) \cdot (-x+y)(-x+y)$

Reading Laws 1 - 3

both ways

#1

$$a^1 = a$$

examples.

$x^2 \cdot y^1 \cdot z^3$

$f(x) = x^1 + 2$

$g(x) = x^2 + 3$

#2

$$a^0 = 1$$

e.x. simplify

$10^0 = 1$

$10^1 = 10$

$(-2)^0 = 1$

$-2^0 = -1 \cdot 2^0$

$= -1 \cdot (1)$

$= -1$

$(x+3)^0 = 1$

$-x^0 = -1 \cdot x^0$

$= -1 \cdot 1$

$= -1$

#3

$$a^m \times a^n = a^{m+n}$$

e.x. simplify

$$3^2 \times 3^4 = 3^{2+4}$$

$$= 3^6$$

$$10^2 \times 10^1 = 10^{2+1} = 10^3$$

$$4^{-2} \times 4^2 = 4^{-2+2} = 4^0 = 1$$

$$a^3 \times b \times a^2 \times b^2 = a^3 \times a^2 \times b^1 \times b^2$$ law #2

$$= a^5 \times b^3$$

You do - Think / Pair / Share

simplify using the laws (verify w calculator)

#1 $4^2 \times 4$ $= 4^3$

#2 $-10^5 \times 10^{-5}$ $= -1 \cdot 10^5 \times 10^{-5} = -1 \cdot 10^0$ $= -1$ $= -1 \cdot 1$

#3 $(-2)^2 \times (-2)^4 = (-2)^6$

#4 $(x^3 y^{-1})(x y^5 z) = (x^4 y^{-1+5} z)$

$= x^4 y^4 z$

Prove : $a^m \times a^n = a^{m+n}$ L.S. R.S.

(Rewriting)

abstractly

L.S.

$a^m \times a^n$

$= a \times a \times \cdots \times a \times a \times a \times \cdots \times a$

m times n times

m + n times

$= a^{m+n}$

R.S. ▨

$a^m \times a^n = a^{m+n}$ ✓

concretely

$a \times a \times a \times a \times a$

3 2

5 times

3 + 2

Prove #2: $a^0 = 1$

start

$a^0 \cdot a^1 = a^{0+1}$ law 3

$a^0 \cdot a^1 = a^1$ law 1

$\frac{a^0 \cdot a}{a} = \frac{a}{a}$

$a^0 = 1$ ☒

You do: Work in same partners

Simplify

i. $2^7 \times 2^{10} = 2^{17}$

ii. $(-3)^4 \times (-3)^3 = (-3)^7$

iii. $-3^4 \times 3^3 = -3^7$

iv. $(xy^2)(x^2y) = (x^3y^3)$

v. $a^2b^3c^4 \times a^3b^1c^2 = a^5 \cdot b^4 \cdot c^6$

vi. $2^7 + 2^{10} \neq 2^{17}$ $(2^7 + 2^7 = 2 \cdot 2^7 = 2^8)$

vii. $(3x)(2x) = 3 \times 2 \times x^1 \times x^1 = 6x^2$

viii $(4x^{-2})(5x^6) = 20x^4$ $(= 4 \times 5 \times x^{-2} \times x^6 = 20x^{-2+6})$

ix $(3a^2)(4b^4) = 12a^2b^4$

Reading Laws 4 and 8-9

#4

$$\frac{a^m}{a^n} = a^{m-n}$$

ex.

$$\frac{4^3}{4^2} = 4^{3-2}$$

$$= 4^1$$

$$= 4$$

ex

$$\frac{y^3}{y^{-3}} = y^{3-(-3)}$$

$$= y^6$$

ex

$$\frac{x^4}{x^4} = x^{4-4}$$

$$= x^0$$

$$= 1$$

Prove: $\frac{a^m}{a^n} = a^{m-n}$ L.S. R.S

L.S.

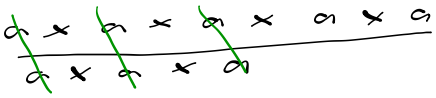
$$\frac{a^m}{a^n}$$

$$\frac{\overbrace{a \times a \times \cdots \times a}^{m \text{ times}}}{\underbrace{a \times a \times \cdots \times a}_{n \text{ times}}}$$

$$\overbrace{a \times a \times \cdots \times a}^{m-n \text{ times}}$$

$$a^{m-n}$$

R.S. ✓ Q.E.D



$5 - 3$

Law #8

$a^{-n} = \frac{1}{a^n}$ - or - $a^{-n} = \left(\frac{1}{a}\right)^n$

ex. $\frac{a^{-2}}{2^1} = \frac{a^{-2}}{1} \cdot \frac{1}{2}$

$= \frac{1}{a^2} \cdot \frac{1}{2}$

$= \frac{1}{2a^2}$

$= \frac{1}{2a^2}$

Recall: x fractions.

ex. $\frac{2}{3} \times \frac{4}{7}$

$\frac{2 \times 4}{3 \times 7}$

ex $\frac{2^{-3}}{1}$

$\frac{1}{2^3}$

ex. 4^{-2}

$\frac{1}{4^2}$

$\frac{1}{4 \times 4}$

$\frac{1}{16}$

ex -3^{-2}

$\frac{1}{-3^2}$

$\frac{1}{-3 \times 3}$

$\frac{1}{-9} = \frac{-1}{9} = -\frac{1}{9}$

Law # 9

$$\frac{1}{a^{-n}} = a^{n}$$

e.x. $\frac{1}{4^{-2}}$

4^{2}

4×4

16

ex $\frac{x^{-3}}{y^{-2}}$

$\frac{y^{2}}{x^{3}}$

Rewrite w/ out negative exponents

e.x. $\frac{a^{3}}{b^{-2}}$

$= \frac{b^{2}}{a^{-3}}$

$= b^{2} \times a^{3}$

$= a^{3} b^{2}$

You: Evaluate / Rewrite as exp # Simplify and final answer can't have negative exponents

i. $3^4 \div 3^2 = 3^2$

ii $3^2 \div 3^4 = \frac{1}{3^2}$ $\left(3^{2-4} = 3^{-2} = \frac{1}{3^2}\right)$

iii. $\frac{5^{10}}{5^4} = 5^{10-4} = 5^6$ #4

iv. $\frac{5^4}{5^{10}} = \frac{1}{5^6}$ $\left(5^{4-10} = 5^{-6} = \frac{1}{5^6}\right)$ #8

v. $3^2 \times 3^{-4} = \frac{1}{3^2}$ $\left(3^{2+(-4)} = 3^{-2} = \frac{1}{3^2}\right)$

vi. $(a^3 b^6) \div (a^3 b^8) = b^{14}$ $\frac{a^3 b^6}{a^3 b^8} = b^{6-8} = b^{-2} = \frac{1}{b^2}$

vii. $\frac{x^2 y^4}{x^1 y^5}$ #4 #4 $= \frac{x}{y}$ $\left(x^{2-1} \cdot y^{4-5} = x y^{-1} = \frac{x}{y}\right)$

viii. $\frac{4x^3}{8x^5} = \frac{1 \cdot x^{3-5}}{2} = \frac{1 \cdot x^{-2}}{2} = \frac{1}{2 \cdot x^2}$

ix $(2x^3)(4x^{-5}y) = 8x^{3+(-5)}y$ #3

$= 8x^{-2} \cdot y$

$= \frac{8 \cdot y}{x^2}$ #8

Reading #5

$$(a^m)^n = a^{m \times n}$$

e.x. $(2^3)^4 = 2^{3 \times 4} = 2^{12}$

e.x. $-(4^2)^5 = -1 \cdot (4^{10}) = -4^{10}$

base of exp 2 : 4

base of exp 5 : 4^2

$-1 \cdot (4^2)^5$

BEDMAS

e.x. $((-5)^3)^2 = (-5)^{3 \times 2} = (-5)^6$

base : -5 of exp 3

base of exp 2 : $(-5)^3$

Law # 6

$(ab)^n = a^n b^n$

↳ base of n = $a \cdot b$

$(a+b)^n \neq a^n + b^n$

e.x.

$(2x^2y^6)^2$

$2^2 \cdot (x^2)^2 (y^6)^2$

$4 x^{2\times 2} y^{6\times 2}$

$4 x^4 y^{12}$

ex. $(x+2)^2$

Can't use law # 6

$(x+2)(x+2)$

$\neq x^2 + 2^2$

ex $(2x)^2$

$(2\cdot x)(2\cdot x)$

$2 \cdot 2 \cdot x \cdot x$

$2^2 x^2$

Law # 7

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

ex. $(xy)^2 = x^2 y^2$

ex. $(x^2 \cdot y^3 \cdot z)^3$ # 6

$= (x^2)^3 \cdot (y^3)^3 \cdot z^3$ # 5

$= x^{2\times 3} \cdot y^{3\times 3} \cdot z^3$

$= x^6 y^9 z^3$

ex. $\left(\frac{x^2}{y^{-3}}\right)^3$ # 7

$\frac{(x^2)^3}{(y^{-3})^3}$ # 5

$\frac{x^{2\times 3}}{y^{-3\times 3}}$

$\frac{x^6}{y^{-9}}$ # 9

$x^6 y^9$

TIPS: • follow BEDMAS
• deal w negative exps @ end

You do:

Simplify and answer must not have negative exponents

i. $(x^{-3}z^{-4})^{-2} = (x^{-3})^{-2}\cdot(z^{-4})^{-2} = x^{6}z^{8}$ #6

ii. $(-2x^2y)^4 = (-2)^4x^8y^4$ $(= (-2)^2(x^2)^4y^4)$ #5

iii. $(a^{-2} \div b^3)^2 = (a^{-2})^2 \div (b^3)^2 = a^{-4} \div b^6$ #7 #8

$= \frac{a^{-4}}{b^6} = \frac{1}{a^4b^6}$

iv. $\left(\frac{4}{x^2}\right)^{-2} = \frac{4^{-2}}{(x^2)^{-2}} = \frac{4^{-2}}{x^{-4}} = \frac{x^4}{4^2}$ #7 #5 #7/#8

v. $\left(\frac{x^3y^4}{4z^{-1}}\right)^2 = \frac{(x^3)^{-2}(y^4)^{-2}}{4^{-2}\cdot(z^{-1})^{-2}} = \frac{x^{-6}y^{-8}}{4^{-2}z^2} = \frac{4^2}{x^6y^8z^2}$

vi. $\left(\frac{x^{-3}}{z^{-4}}\right)^{-2} = \frac{(x^{-3})^{-2}}{(z^{-4})^{-2}} = \frac{x^6}{z^8}$

vii. $\left(\frac{x^4}{x^{-2}}\right)^3 = (x^{4-(-2)})^3 = (x^6)^3 = x^{18}$

viii. $\left(\frac{x^5}{x^2y^3}\right)^{-1} = \left(\frac{x^{5-2}}{y^3}\right)^{-1} = \left(\frac{x^3}{y^3}\right)^{-1}$

$= \frac{x^{-3}}{y^{-3}}$

$= \frac{y^3}{x^3}$

HMWK
P240
and handout