Lesson 7: Multiplication Principle w/out Replacement and Geometric Probability

May 9, 2023

1.9 example

1.9 Example:

You are deciding on a password that is six characters in length. The first two characters must be numbers and the other characters must be letters. Determine the number of possible passwords if ...

a) the password may contain identical characters:

· w/ replacement (yesterday)

$10 \times 10 \times 26 \times 26 \times 26 \times 26$

L L L L

45 697 600 possible outcomes.

b) the password may NOT contain identical characters:

· w/out replacement

$10 \times 9 \times 26 \times 25 \times 24 \times 23$

L L L L

32 292 000 possible outcomes.

step 1 step 2

2 0 2 0 = 4

· use multiplication principle:

(# of possible outcomes in step 1) x (# of possible outcomes in step 2 ...

1.7 Example:

How many ways are there to arrange the letters *ABCD*? That is, what's the number of possible permutations of the four letters?

4 x 3 x 2 x 1 = 24 possible arrangement of letter ABCD.

(A)

4! = 4 x 3 x 2 x 1 = 24

5! = 5 x 4 x 3 x 2 x 1 = 120

6! = 6 x 5 x 4 x 3 x 2 x 1 = 720

n! x!

1.8 Example:

A small independent bookstore received 22 new books and wants to put 4 of them on the main bookshelf for display. How many different possible display arrangements of books are there? That is, what's the number of possible permutations of the four books that is subset of the 22 books?

22! x nor 4! x 18

Display: 22 x 21 x 20 x 19 = 175 560 possible arrangements of the books

1st book

You do

1.10 Practice

34 people compete for the Olympic prize in a swimming competition. The three top performers are awarded gold, silver, and bronze medals. How many different arrangements of final medalists are there?

1.11 Practice

How many different ways can we arrange the numbers 0, 1, 2, 3, 4, 5? That is, what's the number of possible permutations of the 6 numbers?

1.12 Practice E.K.

We need to create a unique 4-digit code where no single number is used more than once. Moreover, the first digit cannot be 0. How many possible 4-digit codes are there?

no replacement/no repetition

option 1st : 1, 2, 3, 4, 5, 6, 7, 8, 9

option 2nd : 0, 1, 2, 3, 4, 6, 7, 8, 9

$9 \times 9 \times 8 \times 7$

$= 4536$ possible passwords

10 x 2 digits

1.13 Practice

Consider the following license plate:

Assuming the first three characters must be numbers and the last three characters must be letters, how many different license plate arrangements are there if no letter nor number can be repeated?

Geometric Probability in One Dimension

Discovery Let's try to solve a problem involving probability and geometric concepts in one dimension and discover a general way to tackle these kinds of questions! Consider the following problem:

What is the probability of randomly selecting a point between 0 and 2?

Recall:

$P(E) = \frac{\#\text{ of fav. outcomes}}{\text{total \# of outcomes}}$

$P(\text{red length}) = \frac{\text{length of red}}{\text{total length}}$

$$P(\text{target}) = \frac{\text{length of target}}{\text{total length}}$$

length / distance:

$d = |x_2 - x_1|$

duration:

coordinates.

$P(\text{red length}) = \frac{2}{3}$ ← fraction

$\frac{66}{100}$

ex. 66 hundredth $= 0.66$ ← decimal

ex. 1 fourth $= \frac{1}{4} = 66.7\%$ ← a percent

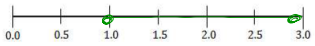
1.1 Example

What is the probability of randomly selecting a point:

(a) between 1 and 3

(b) between 2 and 2.5

(c) between 0 and 3



$d = 3 - 1$

$d = 2$

a) $P(1 \text{ to } 3) = \frac{\text{length from 1 to 3}}{\text{total length}}$

$P(1-3) = \frac{2}{3}$

$= 0.66\overline{7}$

$= 66.7\%$

b) $P(2 \text{ to } 2.5) = \frac{\text{length}_{2-2.5}}{\text{total}}$

$= \frac{0.5}{3} = \frac{1}{6}$ ✓

$= 16.6\%$ ✓

$= 0.166$

c) $P(0 \text{ to } 3) = \frac{3}{3}$

$= 1$

$= 100\%$ certain

$= 0.667 \times 100\%$

$= 0.667 \times \frac{100}{100}$

$\frac{\frac{1}{2}}{3}$

$\frac{1}{2} \div 3$

$\frac{1}{2} \times \frac{1}{3}$

$\frac{1}{6}$

length

perimeter

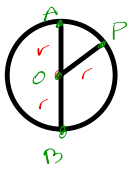
square

$P = s + s + s + s$

$P = 4s$

circumference of a circle:

$C = 2\pi r$, r = radius



$r = \overline{OP} = \overline{OB} = \overline{OA}$

d = diameter

$d = \overline{AB}$

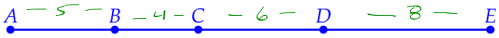
$d = 2r$

You do

→ 1.2. Practice

→ if done, you could challenge yourself in 2D.

1.2 Practice



Given the following:

- $m\overline{AB} = 5$
- $m\overline{BC} = 4$
- $m\overline{CD} = 6$
- $m\overline{DE} = 8$

total length = 5 + 4 + 6 + 8
= 23 units

P(target) = target length / total length

Determine the probability that a randomly selected point lies:

(a) between $\overset{\frown}{A}$ and B (points) P(b/w A and B) = $\frac{5}{23}$ = 0.217 or 21.7%

(b) between A and D P(b/w A and D) = $\frac{5+4+6}{23} = \frac{15}{23}$ = 0.652 or 65.2%

(c) between D and E P(b/w D and E) = $\frac{8}{23}$ = 0.348 or 34.8%

(d) between B and D P(b/w B and D) = $\frac{4+6}{23} = \frac{10}{23}$ = 0.435 = 43.5%

Geometric Probability in 2-Dimension

more specifically: area questions

Discovery Let's try to solve a problem involving probability and geometric concepts in two dimensions and discover a general way to tackle these kinds of questions! Consider the following problem:

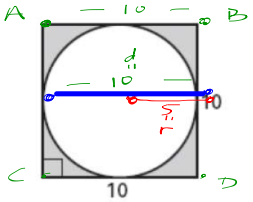
What is the probability of randomly selecting a point in the shaded area?

$A_{square} = s^2$

$A_{rect} = l \cdot w$

$A_{trap} = \frac{(b+B)\times h}{2}$

$A_{circle} = \pi r^2$



draw
· diameter
· radius

$d = 2r$

$\frac{d}{2} = \frac{2r}{2}$

$r = \frac{1}{2}d$

$r = \frac{1}{2}(10)$

$r = 5$

$P = \frac{\#\text{ of fav. outcomes in event}}{\text{total \# of outcomes in } S}$

$P = \frac{\text{target length}}{\text{total length}}$

$P = \frac{\text{target area}}{\text{total area}}$

$P_{\text{shaded region}} = \frac{\text{(1) area of event (SR)}}{\text{(2) area of reference figure}}$

(2) Easiest:

Find square ABCD

$A_S = s^2$

$A = 10^2$

$A = 100\ \text{unit}^2$

(1) Find area shaded Region

$A_{SR} = A_{square} - A_{circle}$ (3)

(3) Find A circle

$A_C = \pi r^2$

$= \pi(5)^2$

$= 25\pi\ \text{units}^2$

$\approx 78.54\ \text{unit}^2$

$A_{SR} = 100 - 25\pi$

$A_{SR} = 21.46\ \text{unit}$

Probability:

$P_{SR} = \frac{\text{Area SR}}{\text{total Area}}$

$P_{SR} = \frac{(100 - 25\pi)}{100}$

$P_{SR} = 0.2146$

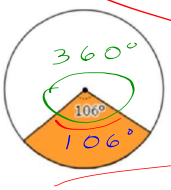
or

$P_{SR} = \frac{21.46}{100}$

$P_{SR} = 0.2146$

$P_{SR} = 21.46\%$

2.1 Example

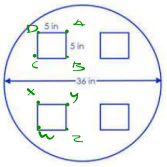


What is the probability of randomly selecting a point in the shaded region?

$P = \frac{106}{360} = 0.2944$

29.44%

2.2 Example



What is the probability of randomly selecting a point in any of the squares?

note: the squares are congruent.

$P = \frac{\text{area target}}{\text{total area}}$

$P = \frac{4\,(A_{squares})}{A_{circle}}$

$d = 36$ inches

$\therefore r = \frac{1}{2}d$

$r = 18$

Find

$A_{square} = 5^2$

$A = 5^2$

$A = 25$ units^2 ✓

Find

$A_{circle} = \pi r^2$

$= \pi (18)^2$

$= 1017.88$ ✓

$P = \frac{4(25)}{1017.88}$

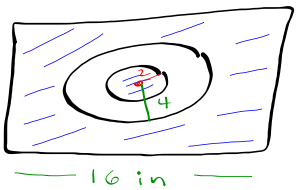
$P = 0.0982$

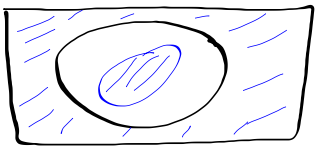
$P = 0.09.82$

$P = 9.82\%$

100%

2.3 Example





What is the probability of randomly throwing a dart and landing on the yellow space?

$$P = \frac{\text{Area of Shaded Region}}{\text{total Area}}$$

$$P = \frac{A_{square} - A_{Big\ circle} + A_{small\ circle}}{A_{square}}$$

$$P = \frac{s^2 - \pi r_B^2 + \pi r_s^2}{s^2}$$

$$P = \frac{(16^2 - \pi(4)^2 + \pi(2)^2)}{16^2}$$

$$P = 0.8527$$

$$P = 85.27\%$$

You do:

- 2.6 Practice (not 2.5)
- 2.7 Practice

} 5 min after lunch for questions.

- Cuz must start tasks!

Task: Push Your Luck

Random Experiment:

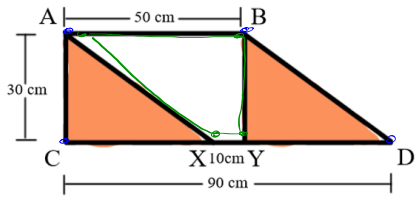
You play the following 2-step game of chance at a fair.

Step 1: Drawing a Marble from a Bag

A marble is drawn from a bag containing orange and blue marbles.
The probability of drawing a blue marble is 0.2.

Step 2: Shoot a Dart at a Target

A dart is thrown at trapezoid ABCD that is composed of smaller trapezoid XYAB and congruent right triangles ACX and BYD. It is assumed that you will hit the target.



To win the game, you must shoot the dart in the shaded region and draw an orange marble.

What is your probability of winning?

Tree Diagram

- step 1: draw a marble
- step 2: shoot a dart
- Outcomes

8/10 = 0.8 (O) × White Region (O, WR) 0.5714 = winning; S.R. (O, S.R.)

equal side, equal area

B: 0.2 = 2/10 × W.R. (B, WR); S.R. (B, S.R.)

× times; Probabilities (horizontal)

Step ① Find prob of drawing orange marble.

Event Or. mar. is the complement of the event bl. marble.

$P(A) + P(A') = 1$

$P(A) = 1 - P(A')$.

$P(Or.\ mar) = 1 - P(bl.\ mar)$

$P(O) = 1 - 0.2$

$P(O) = 0.8$

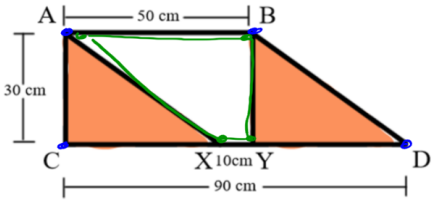
$P(O) = \frac{1}{1} \times \frac{10}{10} - \frac{2}{10}$

$= \frac{10-2}{10}$

$P(O) = \frac{8}{10}$

$P(Or.\ mar) = \frac{\#\ of\ fav\ outcomes}{total\ outcomes}$

② Find Probability of shaded Region



$P(S.R.) = \frac{area\ target\ (S.R.)}{total\ area}$

① $A_{S.R.} = A_{trap\ ABCD} - A_{trap\ AXYB}$

$A_{S.R.} = \frac{(b+B)\times h}{2} - \frac{(b+B)\times h}{2}$

$= \frac{(50+90)\times 30}{2} - \frac{(10+50)\times 30}{2}$

$A_{S.R.} = 1200\ units^2$

② $A_T = A_{trap\ ABCD}$

$= \frac{(50+90)\times 30}{2}$

$A_T = 2100\ units^2$

③ $P = \frac{1200}{2100}$

$P = \frac{12}{21}$

$= 0.5714$

$= 57.14\%$

④ To win, We want orange marble AND shaded Region

$P(WIN) = P(or\ mable) \times P(S.R.)$

$= 0.8 \times 0.5714$

$= 0.45712$ or 45.7% chance of winning!

Hmwk
p 161 just a) - c)
~~p 201~~
#4.14
p 178 - 179