Lesson 6: Probability Definitions and Tree Diagrams

May 8, 2023

Recall: Definition:

Probability → a measure (a decimal/a percent/a fraction) that describes how probable/likely it is that an event will occur/happen, especially an event based on chance, based on past performance.

0.5 50% $\frac{1}{2}$

« le hasard » « az zahr »

event A = rolling a '6'
↳ theoretical probability

event B = serving a meat eater @ pizza store.
↳ experimental probability

Random Experiment: a process/game for which the outcomes cannot be predicted w/ certainty.

e.x. $R.E._1$: Rolling a six-sided die. || $R.E._2$: Serving a pizza customer.

· Universe / sample space of R.E. → all possible outcomes of the R.E.
Ω (omega)

{ } - listing

ex.
$\Omega_{RE_1} = \{1, 2, 3, 4, 5, 6\}$ || $\Omega_{RE_2} = \{$veggie, meat, pinapple$\}$

· Event of R.E. (subset of Ω)
↳ when a specific outcome of R.E. is specified.

$R.E._1$

Event A = Rolling a '1'

Event A = $\{1\}$

the probability of event A:

$P(A) = \frac{1}{6}$

Event B = {Rolling an even #}

Event B = $\{2, 4, 6\}$

$P(B) = \frac{3}{6} = \frac{1}{2}$
$= 50\%$
$= 0.5$

||

$R.E._2$

Event C = Serving a veggie customer

→ we need past data to answer about prob.

Representing a R.E. w̄ a Tree Diagram (graph)

P 132 - 133   R.E. : Flipping a coin 3 times

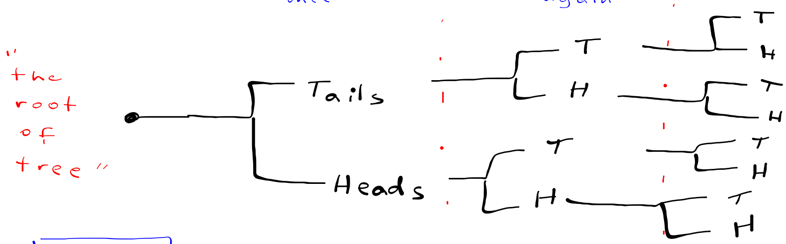
↳ always break into steps.

· step 1 : · 2 possible outcomes "flip once"

step 2 : · 2 possible outcomes "flip again"

step 3 : · 2 possible outcomes "flip a 3rd time"

Outcomes:



(T, T, T)
(T, T, H)
(T, H, T)
(T, H, H)
(H, T, T)
(H, T, H)
(H, H, T)
(H, H, H)

sample space Ω (ordered triples)

P 134 : How to calculate the total # of possible outcomes of R.E.

Use : Multiplication Principle for R.E. w̄ n steps.

ex. $2 \times 2 \times 2 = 8$ possible outcomes

(# of possible outcomes in step 1) × (# of p.o. in step 2) × ... (# of p.o. in step n)

N.B. — no tree for R.E. w̄ more than 3 steps

When $\boxed{\text{M.P.}}$ is better than Tree Diagram:

1.1 Example

You play a game where you first draw a card from a 52-card deck and then roll a six-sided die. How many possible outcomes are there?

- step 1
- draw a card

step 2
- roll die

= total possible outcomes

52 p.o. × 6 p.o. = 312 possible outcomes.

1.2 Example

You flip a coin, roll an eight-sided die and then roll a twenty-sided die. How many possible outcomes are there?

step 1 step 2 step 3

2 p.o. × 8 p.o. × 20 p.o. = 320 p.o.

1.3 Example : E.K.

You are deciding on a password that is six characters in length. The first two characters must be numbers and the other characters must be letters. How many possible passwords are there?

- 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- 26 letters in alphabet

$\underset{\#}{\underline{10}} \times \underset{\#}{\underline{10}} \times \underset{L}{\underline{26}} \times \underline{26} \times \underline{26} \times \underline{26}$

45 697 600 possible passwords

1.4 Practice

You do 1.4 - 1.7

Takisha and Laurence each flip a coin. How many possibilities are there?

4.

1.5 Practice

Takisha, Laurence and Bar each roll a six-sided die. How many possibilities are there?

T L B

6 × 6 × 6 = 216

1.6 Practice

You are deciding on the menu for a cafeteria and must choose from three appetizer options, 4 main meal options, 2 desserts and 3 drinks. How many different meal combinations are there?

72

1.7 Practice

Consider the following license plate:

Assuming the first three characters must be numbers and the last three characters must be letters, how many different license plate combinations are there?

17 576 000

$\underline{10} \times \underline{10} \times \underline{10}$ ✓ $\underline{26} \times \underline{26} \times \underline{26}$

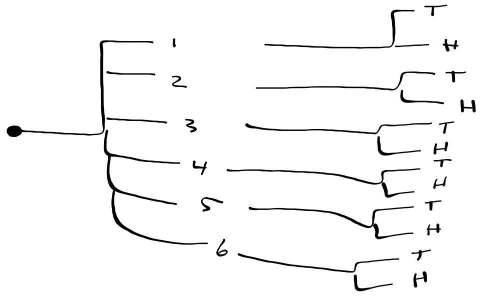
You draw a Tree for R.E:

→ You roll a 6-sided die and flip a coin:

step 1 roll die
• 6 possible outcomes

step 2 • flip coin
• 2 possible outcomes

Outcomes



(1, T)
(1, H)
(2, T)
(2, H)
(3, T)
(3, H)
(4, T)
(4, H)
(5, T)
(5, H)
(6, T)
(6, H)

P 139

How to calculate the probability of Event

$$\boxed{P(E) = \frac{\#\text{ of outcomes in event }(E)}{\text{total }\#\text{ of outcomes (in R.E./}\Omega)}}$$

$P(E) = 0$

• An Impossible event

ex. Roll a 8 and flip a tail

$P(E) = \frac{0}{12}$

$P(E) = 0$

$0 < P < 1$

• A probable event

ex Event B = roll a 6

Event B = {(6, T), (6, H)}

$P(B) = \frac{2}{12}$

Event C: flip a tails

{(1, T), (2, T), (3, T), (4, T), (5, T), (6, T)}

$P(C) = \frac{6}{12}$

$= \frac{1}{2}$

$= 50\%$

$= 0.5$

$P(E) = 1$

$= 100\%$

$= \frac{12}{12}$

Event D

= {roll 1, 2, 3, 4, 5 or 6 and flip T or H}

$P(E) = \frac{12}{12}$

A Certain Event

2.1.1 Example

probable

Consider the following events and determine whether they are *possible*, *impossible* or *certain*:

(a) Rolling a six-sided die and landing on an even number.

R.E. event

possible/probable.

(b) Rolling an eight-sided die and landing on a number greater than 8.

R.E. event

: impossible $P = 0$

(c) Rolling a six-sided die and landing on a number that is both even and odd.

. impossible

(d) Rolling a six-sided die and landing on a number that is either even or odd.

R.E. event

$P = \frac{6}{6} = 1$ $\boxed{\text{certain}}$

$\Omega = \{1, 2, 3, 4, 5, 6\}$

$E = \{2, 4, 6, 1, 3, 5\}$

(e) It will snow in July.

. possible

(f) It will both rain and not rain at the same time tomorrow.

. impossible

(g) It will either rain or it will not rain tomorrow.

. certain

You do practice

2.1.2 – 2.2.1

check answers.

$\boxed{\text{p 10 } \quad 2.2.3 : \text{Bonus.}}$

2.1.2

f) · possible

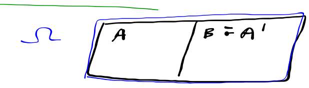
0.0000001%

e)   impossible

find a shape that's both a square and a rectangle.

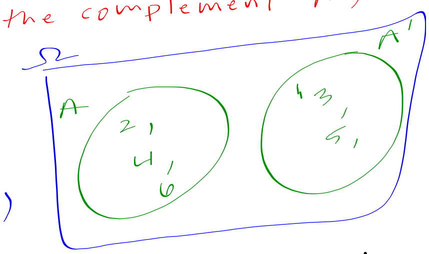
possible

Describing Two Events:



Definition: Complementary Events are incompatible events (no outcomes in common / mutually exclusive) whose outcomes make up the entire sample space Ω

N.B.: The complementary event to A is not - A (or A')
(A prime)
(not - A is the complement A)



Example: Find the complement.

i. R.E. Roll a 6-sided die

Event A : Rolling an even #

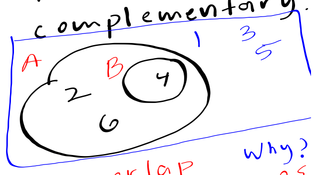
Event A' : Rolling an odd #
(not - A) (not an even #)

complementary

Event B : Rolling a 4

Event B' : Rolling not - a 4
(not B) Rolling a 1, 2, 3, 5, 6

Q: Is event A and B complementary? NO



Why?
· overlap
· more outcomes in R.E

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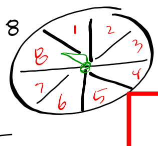
ii R.E. Spinning a 3, 4 or 6 (Event C:) on a spinner marked 1-8.

Event C' = spinning 1, 2, 5, 7, 8
(not C)
(not 3, 4, or 6)



$P(C) = \frac{3}{8}$ $\quad P(C') = \frac{5}{8}$

$$\boxed{P(E) = \frac{\#\text{ of outcomes in event}}{\text{total }\#\text{ of outcomes in }\Omega}}$$

$$\boxed{P(C) + P(C') = 1}$$

→ where C and C' are complementary events.

∴

$$\boxed{\begin{gathered} P(C') = 1 - P(C) \\ \text{or} \\ P(C) = 1 - P(C') \end{gathered}}$$

solve/isolate

$\cancel{P(C)} + P(C') = 1 - P(C)$
$\cancel{-P(C)}$

$P(C') = 1 - P(C)$

handy:

Draw a card out of 52-deck.
What's the prob the card is not 2

$P(\text{not } 2) = 1 - P(2)$

$$\boxed{P = \frac{\#\text{ of out}}{\text{total}}}$$

$P(\text{not } 2) = \frac{1^{\times 52}}{1 \times 52} \quad \frac{4}{52}$

$P(\text{not } 2) = \frac{48}{52}$

You do 2.4.1 / 2.4.3.

2.4.1 Example

A bag contains 5 white, 3 black, and 2 red balls. Determine the probability that the ball is not red.

$$P(\text{not red}) = 1 - P(\text{red}) = 1 - \frac{2}{10} = \frac{8}{10}$$

$$P = \frac{\#\text{ of outcomes}}{\text{total}}$$

2.4.2 Practice

When tossing a fair standard die, determine the complement of rolling a number less than 2.

$$P(\text{not less than } 2) = 1 - P(\text{less than } 2) = 1 - \frac{1}{6} = \frac{5}{6}$$

2.4.3 Practice

What is the probability you will not get a sum of 6 if the the probability of getting a sum of 6 when rolling two dice is 5/36?

$$P(\text{not sum}) = 1 - P(\text{sum}) = 1 - \frac{5}{36} = \frac{31}{36}$$

<< the house always winss
- not fair
- gates of hells
→ casnio craps
(unfair man)
→ shooting dice ✓

Rules of Casino Craps:

1st Roll:

Event Win:
- if you roll 7 or 11.

Event Lose:
- if you roll 2, 3 or 12.

y | n

Are they complementary? NO.

Event Stop Rolling
- if you win or lose

if 7, 11, 2, 3, 12

Event Do Not Stop Rolling

e.x. a 6

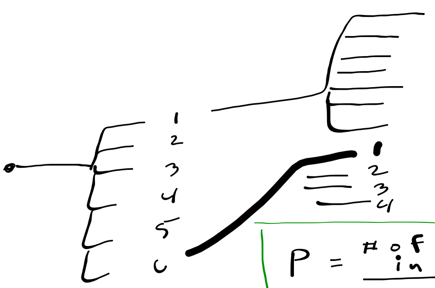
↳ if you get any other # in S other than 7, 11, 2, 3, 12

→ that's your point.

step i: Roll one die

step ii: Roll 2nd die

6 p.o. × 6 p.o = 36 p.o.



(6, 1) = 7
(6, 2) =

$$P = \frac{\text{\# of outcomes in event}}{\text{total \# outcomes}}$$

outcomes

(1, 1) = a 2
(1, 2) = a 3
(1, 3) = a 4
(1, 4) = a 5
(1, 5) = a 6
(1, 6) = a 7

(2, 1) = a 3
(2, 2) = a 4
(2, 3) = a 5
(2, 4) = a 6
(2, 5) = a 7
(2, 6) = a 8

(3, 1) = a 4
(3, 2) = a 5
(3, 3) = a 6
(3, 4) = a 7
(3, 5) = a 8
(3, 6) = a 9

1 Roll

- Event Win
  - a 7 $= \frac{6}{36}$
  - or a 11 $= \frac{2}{36}$
  - $+$ $\frac{8}{36} = 22\%$

Event Lose
- a 2 $P = \frac{1}{36}$
- or 3 $P = \frac{2}{36}$
- or 12 $= \frac{1}{36}$
- $+$ $\frac{4}{36} = 11\%$

Event Keeping Rolling

Event (Not win nor Lose)

$P(\text{N-W/L}) = 1 - P(\text{W or L})$

$= 1 - \left(\frac{8}{36} + \frac{4}{36}\right)$

$= \frac{24}{36} = 67\%$

2 Roll

- Event Win
  - your point
  - ex - a '6'

$\boxed{\text{Event Lose} - \text{a '7'}}$

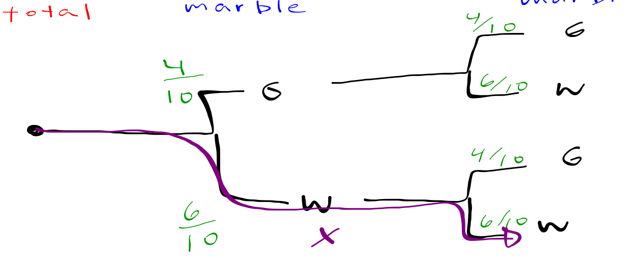
R.E. with and without Replacement

e.x. make tree diagram of the R.E. of drawing 2 marbles from bag $\overline{w}$ replacement) 6 white and 4 green marbles. AND you put marble back in bag

$P = \dfrac{\text{\# of favourable outcomes}}{\text{total}}$

step 1 draw 1st marble

step 2 draw 2nd marble



Outcomes: Probabilities:

(G, G) $\frac{4}{10} \times \frac{4}{10} = \frac{16}{100} = 16\%$

(G, W) $\frac{4}{10} \times \frac{6}{10} = \frac{24}{100} = 24\%$

(W, G) $\frac{6}{10} \times \frac{4}{10} = \frac{24}{100} = 24\%$

(W, W) $\frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = 36\%$

+

$1 = \frac{100}{100} = 100\%$

✓ nota bene = multiply the probs when going horizontally along tree (when you say "and"; example, draw a white and a green)

Event Win = draw 2 marbles of same colour.

$W = \{ (G,G) \text{ or } (W,W) \}$

$P(W) = \frac{16}{100} + \frac{36}{100} = \frac{16+36}{100} = \frac{52}{100} = 52\%$

complementary: $P(\text{Lose}) = 1 - 52\% = 48\%$

✓ nota bene = add the probs when going vertically along tree (when you say "or"; ex. you win if you get (W,W) or (G,G))

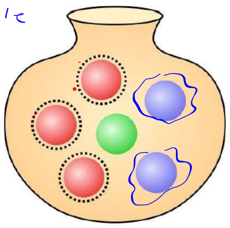
2.2 Example

You are participating in game in which you select two ~~balls~~ marbles in turn from ~~an urn~~ a bag. Two rules are possible: either you replace the ~~balls~~ marble in the ~~urn~~ bag after selection, or you do not. Winning the game involves selecting a green and blue ~~ball~~ marble (in any order).

(a) What is the probability of winning the game *with* replacement?

(b) What is the probability of winning the game *without* replacement?

(c) If we change the winning conditions in each case as follows: winning the game involves selecting a green and blue ball (in that order) or selecting a green and red ball (in that order), how will it affect the probability of winning the game?



You do:

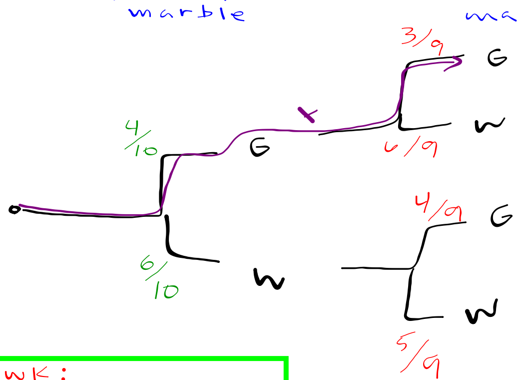
- Make a tree $\overline{w}$ replacement and answer a) &

- Make a tree $\overline{w}$ out replacement and answer b)

R. E. w/out Replacement.

draw 2 marbles (w/out replacement)
from bag containing 6 white and 4 green.

1st step draw 1st marble | 2nd step draw 2nd marble | Outcome



(G, G) $\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$

(G, W) $\frac{4}{10} \times \frac{6}{9} = \frac{24}{90}$

(W, G) $\frac{6}{10} \times \frac{4}{9} = \frac{24}{90}$

(W, W) $\frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$

$\frac{90}{90} = 100\%$

Hmwk:
- finish handouts
- p 128 - 129 (2.a b).
- p 141
- p 144
- p 152