Lesson 4 : Measures of Dispersion and Box and Whisker Plots

ex. Range ex Interquartile Range

May 3-4, 2024

↳ Range
↳ 1 a Range
↳ 4 quarters & 4 individual ranges
↳ 3 quartiles (Q_1, Q_2, Q_3)
(dispersion) md

Quartiles are a way to measure the spread of data as they allow us to measure certain positions. Quartiles split the data into 4 ~~roughly~~ equal parts.

1.1 Example

Consider the following distribution:

{ 12, 33, 56, 78, 19, 99, 44, 55, 66, 17, 8, 40, 33, 48, 17, 23, 22 }

Determine the quartiles, four quarters and *interquartile range* of the data set.

Q_1 / Q_2 / Q_3
md.

Recall: How to find median = Quartile (2)

(1) Position = $\frac{n+1}{2}$ md/Q_2

$= \frac{17+1}{2}$

$= 9^{th}$ position

(2) organize dist. from min to max

min 18 Q_1 55.5 Q_3 max

8, 12, 17, 17, 19, 22, 23, 33, 33, 40, 44, 48, 55, 56, 66, 78, 99

4th 5th Q_2

nota bene:

- Q_1 is median of 1st subgroup.
- Q_3 is median of 2nd subgroup

$P_{Q_1} = \frac{n+1}{2}$ n = # of value in subgroup

$P_{Q_1} = \frac{8+1}{2}$

$P_{Q_1} = 4.5$ → (middle average of 4th and 5th)
4th and 5th

$Q_1 = \frac{17+19}{2}$

$Q_1 = 18$

$P_{Q_3} = \frac{8+1}{2}$

$= 4.5$
4th 5th

min 18 Q_1 55.5 Q_3 max

8, 12, 17, 17, 19, 22, 23, 33, 33, 40, 44, 48, 55, 56, 66, 78, 99

4th 5th Q_2

Find: 3 Quartiles (data values)

$Q_1 = 18$
$Q_2 = 33$
$Q_3 = 55.5$

Find:

Range = max − min

Range = 99 − 8

Range = 91

Interquartile Range = $Q_3 - Q_1$

IR = 55.5 − 18

IR = 37.5

Find: 4 Quarters (sets)

1st Quarter = {8, 12, 17, 17} ~25%

2nd Quarter = {19, 22, 23, 33} ~25%

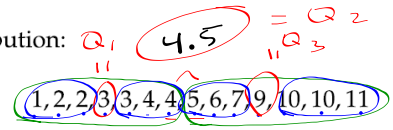
3rd Quarter = {40, 44, 48, 55} ~25%

4th Quarter = {56, 66, 78, 99} ~25%

→ You do Ex 1.2.

1.2 Example

Consider the following distribution:



Determine the quartiles, four quarters and *interquartile range* of the data set.

Q_2 :

$P_{Q_2} = \frac{n+1}{2}$

$P_{Q_2} = \frac{14+1}{2}$

$P = 7.5$

7^{th} & 8^{th}

$P_{Q_1} = \frac{7+1}{2}$

$P_{Q_1} = 4^{th}$ position $= P_{Q_3}$

1st quarter = { 1, 2, 2 }

2nd quarter = { 3, 4, 4 }

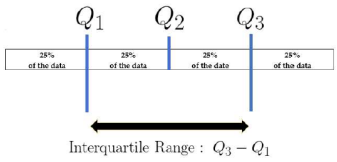
3rd quarter = { 5, 6, 7 }

4th quarter = { 10, 10, 11 }

Quartiles

Quartiles divide the data into four equal groups (quarters):

- Q_1: **First Quartile (Lower Quartile)**. Splits off the lowest 25 % of data from the highest 75%
- Q_2: **Second Quartile (Median)**. Splits the data in half.
- Q_3: **Third Quartile (Upper Quartile)**. Splits off the highest 25 % of data from the lowest 75%



Finding Quartiles

1. Find Q_2 (the median) using $\frac{n+1}{2}$ to find the position of the middle value (n= # of values in the entire data set)
2. Find Q_1 using $\frac{n+1}{2}$ to find the position of the middle value in the data to the left of Q_2 (n= # of values to the left of Q_2)
3. Find Q_3 using $\frac{n+1}{2}$ to find the position of the middle value in the data to the right of Q_2 (n= # of values to the right of Q_2)

Pg 4

Graphical Representation of

- 4 quarters
- 3 quartiles
- Range
- Interquartile Range

2.1 Practice

Fill out the following information:

- The maximum is 8
- The minimum is 0
- The range is 8
- The lower quartile (Q_1) is 2
- The median (Q_2) is 3
- The upper quartile (Q_3) is 7
- The interquartile range is 5

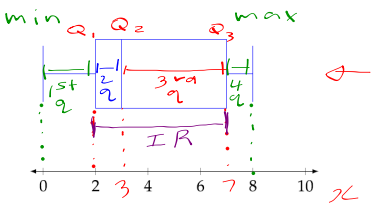
$IR = Q_3 - Q_1$

$= 7 - 2$

$IR = 5$

Box and Whisker Plot:

Range = max - min



← a distribution of x-values rep. b+w plot.

↳ communicates dispersion

Take note: IR is a more interesting measure of dispersion than than the range, since the range sometimes includes outlyer data values.

Exam Results

outlyer = not following the tendency of dis.

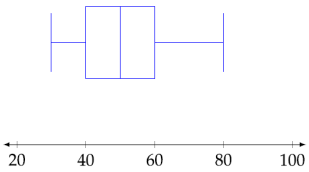
↳ the 3rd quarter is the most dispersed but still contains 25% of data values

You do practise. 2.2. + start reading pg 3 and answer questions.

2.2 Practice

Fill out the following information:

- The maximum is _____
- The minimum is _____
- The range is _____
- The lower quartile (Q_1) is _____
- The median (Q_2) is _____
- The upper quartile (Q_3) is _____
- The interquartile range is _____



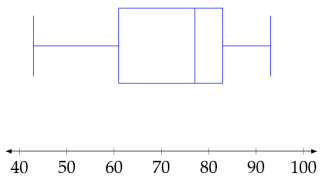
2 Visualizing Data: Box and Whisker Plots

The final grades of Ms. Awolla's Science class were analyzed and the following was noted:

- The highest grade (maximum) is 93
- The lowest grade (minimum) is 43
- The lower quartile (Q_1) is 61
- The median (Q_2) is 77
- The upper quartile (Q_3) is 83

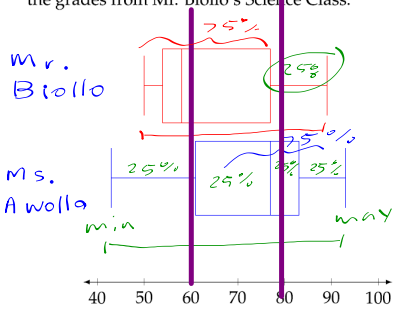
These positions allow us to construct an important visualization called *box and whisker plots* (example on the right). These visualizations show the dispersion of the data.

Grades in Ms. Awolla's Science Class



Read your answers in partner

Box and Whisker Plots are quite useful and allow us to compare distributions quite easily. Consider the following visualization. In addition to Ms. Awolla's Science Class grades we included the grades from Mr. Biollo's Science Class:



1. Which class has a higher range of grades?
   Ms. Awolla
2. In which class did a larger percentage of students pass?
   Ms. Awolla
3. In which class did a larger percentage of students receive a grade below 80%?
   Mr. Biollo ✓

Which quarter in which class is dispersed the least?
Mr. Biollo 1st or 2nd quarter.

Constructing a Box-and-Whisker Plot

Example Construct a box and whisker plot for the following data set:

2, 4, 7, 8, 8, 9, 3, 4, 5, 6, 4, 3, 4, 3, 2, 7, 7

① order values

2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 6, 7, 7, 7, 8, 8, 9

min = 2 max = 9

② Find min, max, Q_2, Q_1, Q_3

$P_{Q_2} = \frac{17+1}{2}$

$P_{Q_2} = 9^{th}$ position

$Q_2 = 4$

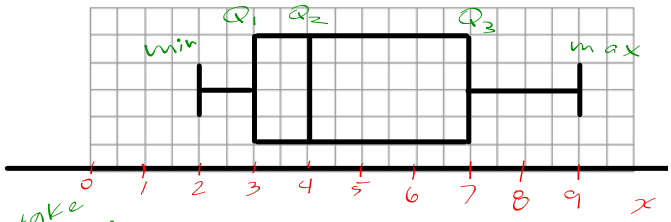
$P_{Q_1} = \frac{n+1}{2} = \frac{8+1}{2} = 4.5$

$4^{th} + 5^{th}$ take average $\frac{3+3}{2}$

$Q_1 = 3$

$Q_3 = 7$

title: (for contexts)



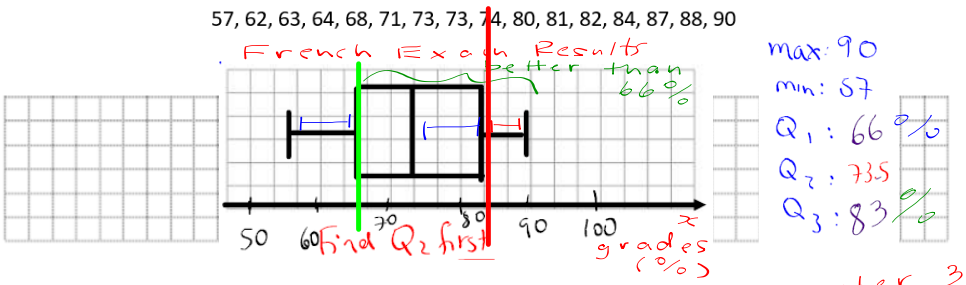
title: (for context)

③ Graduate x-axis, start close below the min, and go up by 1.

you do!

Example The following distribution lists the grades that Adult Education students got on their French exam. Visualize the distribution using a box and whisker plot, and then answer the following questions:

57, 62, 63, 64, 68, 71, 73, 73, 74, 80, 81, 82, 84, 87, 88, 90



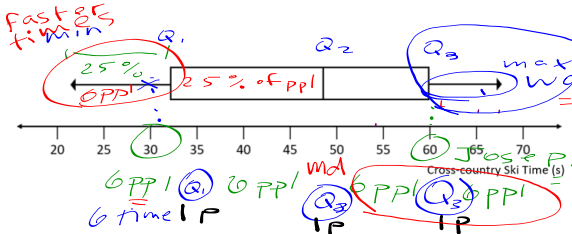
max: 90
min: 57
Q_1: 66%
Q_2: 73.5
Q_3: 83%

Question 1: In which quarter are the data values the most dispersed? quarter 3
Question 2: In which quarter are the data values the least dispersed? quarter 4
Question 3: What percentage of students got a grade better than 66%? 75% of the students
Question 4: What percentage of students got a grade better than 83%? 25%
Question 5: What is the interquartile range? IR = Q3 − Q1 IR = 17
Question 6: What is the range? Range = max − min Range = 33

PRACTICE QUESTIONS: READING AND INTERPRETING BOX-AND-WHISKER PLOTS

1 + 26 = 27 people in race

Question 1: Joseph and 26 other skiers participated in a cross-country ski race. The distribution of their times is represented in the box-and-whisker plot below.



a) How many people got a ski time in between the minimum time and the first quartile?

6 PPl

b) How many people got a ski time higher than the median?

13 ppl

c) Joseph's ski time was 60 seconds. If in a race you want a smaller time, how many people did Joseph perform better than?

6 ppl

d) Another participant got a ski time of 30 seconds and claims her time is among the 6 best times. Is she correct?

Yes she correct since her time is in 1st quarter!

HMWK:

pg 91 #2.19

pg 93 #2.22

pg 99 #2.29