

## LESSON 12: DIFFERENTIATING BETWEEN AN EXPONENTIAL AND QUADRATIC FUNCTIONS

### TASK 1 – A WELL DEFINED PROBLEM IS HALF SOLVED

Modern western societies allow individuals to live more independently than previous generations where community was more fundamental to people's livelihood. Irrespective of the pros and cons, individualistic cultures demand people to be more financially responsible because there are fewer safety nets when people reach retirement age.

The first steps in becoming financially responsible can be as simple as collecting data on your different sources of income and organizing it in such a way to see the bigger picture. For example, James made a chart of how his annual salary grew over the years since 2000. The second chart summarizes his second source of income: end-of-decade bonuses.

Determine James' total income in 2010 versus his projected income in 2020.

step i. Define variables

step ii. Find equation

$$f(x) = ab^x$$

$$f(x) = 40000b^x$$

$$y = 40000b^x$$

step iii find last parameter by subbing in any point in equation

$$\frac{44100}{40000} = \frac{40000b^2}{40000}$$

$$\sqrt{1.1025} = \sqrt{b^2}$$

$$1.05 = b$$

$$40000 = a$$

$$f(x) = ab^x$$

$$f(x) = 40000(1.05)^x$$

initial value not (0,0)  
(0, 40000)  
(0, a)

solve  
0.0.  
B  
D  
M  
A  
S

SOURCES OF INCOMES

| Elapsed Time (years) | Salary (\$) |
|----------------------|-------------|
| 0                    | 40 000      |
| 1                    | 42 000      |
| 2                    | 44 100      |
| 3                    | 46 305      |
| 4                    | 48 620.25   |

| Time (years) | Bonus (\$) |
|--------------|------------|
| 2010         | 5 000      |
| 2020         | 7 000      |

find y when x = 10

find f(10)

$$f(10) = 40000(1.05)^{10}$$

$$f(10) = 65155.79$$

+ 5000 \$ Bonus

∴ In 2010, James makes \$ 70 155.79

find y when x = 20

find f(20)

$$f(20) = 40000(1.05)^{20}$$

$$f(20) = 106131.91$$

+ 7000 \$

∴ In 2020, he makes \$ 113 131.91

**TASK 2 – FIXED COSTS VERSUS VARIABLE COSTS**

Collecting data on one's expenses is more tedious due to the numerous ways we spend our money. There are fixed costs (expenses that are constant over a certain period, e.g. rent, phone bill) and variable costs (expenses that vary depending on our activity, e.g. food, leisure). Nevertheless, collecting data on how you spend your money is imperative to gain control over your finances.

During a couple of months, every time James bought something he wrote down the amount. He then organized his expenses into different categories to understand his spending habits more clearly. He noted that the money he spent on food grew exponentially the more he went out to eat.

EXPENSES

| Number of Times Went out to Eat | Monthly Money Spent at Restaurants (\$) |
|---------------------------------|---|
| 0                               | 0                                       |
| 1                               | 10.50                                   |
| 2                               | 42                                      |
| 3                               | 94.5                                    |
| 4                               | 168                                     |

| Monthly Fixed Costs | Amount (\$) |
|---------------------|-------------|
| Rent                | 770         |
| Internet            | 35          |
| Phone               | 80          |
| Hydro               | 40          |

Using the below charts, determine the total amount of James' monthly expenses if he went out to eat 4 times versus 30 times.

Total exp = (fixed + ~~resto~~ exp)

$$f(x) = ax^2$$

$$y = ax^2 \quad (2, 42)$$

$x \quad y$

$$42 = a(2)$$

(evaluate)

$$\frac{42}{4} = \frac{a(4)}{4}$$

(solve) 0.0.

$$10.5 = a$$

$$f(x) = 10.5x^2$$

$$y = 10.5x^2$$

\$925

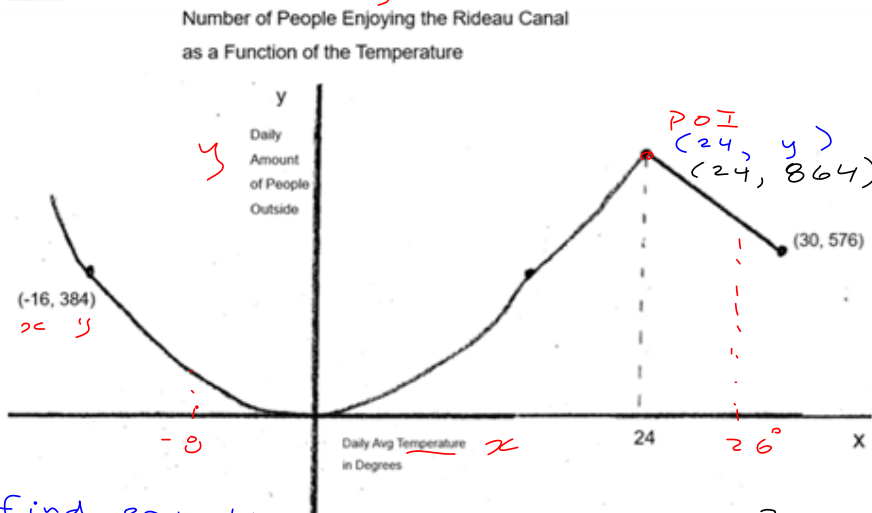
|   |   |
|---|---|
| <p>find y when <math>x=4</math> find <math>f(4)</math></p> $f(4) = 10.5(4)^2$ $f(4) = \$168$ $+ \$925$ $TE = 1093 \$$ | <p>find y when <math>x=30</math> find <math>f(30)</math></p> $f(30) = 10.5(30)^2$ $f(30) = \$9450$ $+ \$925$ $TE = \$10375$ |
|---|---|

**TASK 4 – THE LACHINE CANAL FESTIVITIES**

The city of Montreal noted the number of visitors at the Lachine Canal for the year 2018. They constructed the below graph which shows how the daily amount of people at the canal depends on the outdoor temperature.

This year the city is holding two events that will bring a certain number of people to the Lachine Canal in addition to the average amount of people. Igloofest in December which is expected to attract 500 people and *Le village éphémère* in August that will host around 700 people.

Using the second chart that notes the average monthly temperature, determine the total amount of people there will be at the Lachine Canal during the two events.



| Month | Avg Daily Temp (Cel) |
|-------|----------------------|
| Jan   | -16                  |
| Feb   | -11                  |
| Mar   | 2                    |
| Apr   | 5                    |
| May   | 15                   |
| June  | 24                   |
| July  | 26                   |
| Aug   | 26 = x               |
| Sept  | 17                   |
| Oct   | 13                   |
| Nov   | 0                    |
| Dec   | -8 = x               |

find equation

$$f(x) = ax^2$$

$$384 = a(-16)^2$$

$$\frac{384}{256} = \frac{a(256)}{256}$$

$$1.5 = a$$

$$f(x) = 1.5x^2$$

$$f(-8) = 1.5(-8)^2$$

$$f(-8) = 96 \text{ ppl}$$

+ 500 ppl  
∴ 596 ppl in Dec

$$f(24) = 1.5(24)^2$$

$$f(24) = 864$$

find equation

$$y = ax + b$$

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{576 - 864}{30 - 24}$$

$$a = -48$$

$$y = -48x + b$$

$$864 = -48(24) + b$$

$$864 = -1152 + b$$

$$2016 = b$$

$$y = -48x + 2016$$

$$y = -48(26) + 2016$$

$$y = 768 \text{ ppl}$$

$$+ 700 \text{ ppl}$$

∴ 1468 people in August