Unit 5: The Inverse of a Real Function

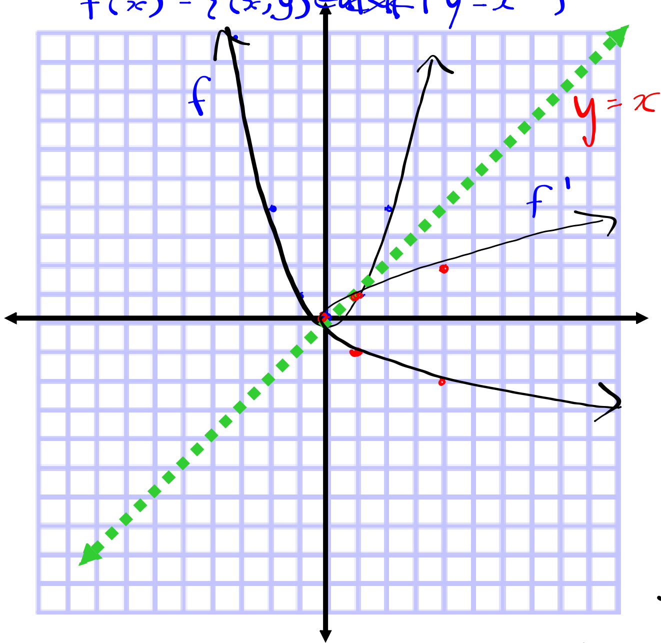
$f(x) = \{(x,y) \in \mathbb{R}\times\mathbb{R} \mid y = x^2\}$

f'

$f'(x)$

f

$f(x)$



Definition: the inverse of a function is its reflection around the $y = x$.

The inverse exchanges/swapes the x and y values of the function.

ex. $f = \{(-1,1), (1,1), (2,4), (-2,4), (0,0)\}$

$f' = \{(1,-1), (1,1), (4,2), (4,-2), (0,0)\}$

Domain f : $\mathbb{R}$

Range f : $[0, \infty$

Domain f' $[0, \infty$

Range f' $\mathbb{R}$

Always true: Domain f = Range f'

Range f = Domain f'

Finding the equation of the inverse of function algebraically:

e.x.
find f^{-1} of

$f(x) = x^2 - 4$

$y = x^2 - 4$

$\underset{+4}{x} = y^2 - 4 \; {}^{+4}$

$\sqrt{y}\,\cancel{x} = \sqrt{x+4}$

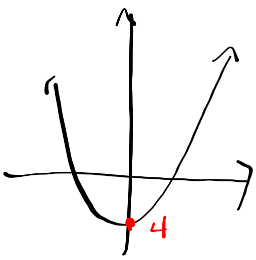
$y = \pm\sqrt{x+4}$

f^{-1}: Domain: $[-4, \infty$
Range: $\mathbb{R}$

Step ①: Switch x and y in the equation

Step ②: Solve for y by isolating.

Step ③: State $y=$ / Switch the Domain and Range



f: Domain: $\mathbb{R}$
Range: $[-4, \infty$

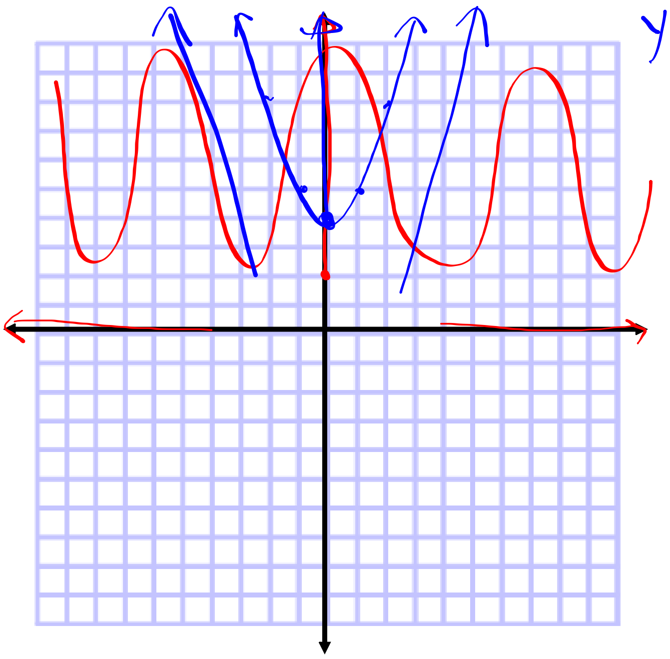
find the inverse:

$f(x) = 4(x-2)^2 + 3$

negative

$f(x) = \{ (x, y) \in \mathbb{R} \times [2, \infty \mid y = x^2 + 4 \}$

$y = x^2 - 4$



find the inverse:

$f(x) = \{(x,y) \in [0,10] \times \mathbb{R} \mid f(x) = 4x - 5\}$

find $f'(2)$

Define the interval over which f' is increasing and negative : none

f' : $y = 4x - 5$

$x + 5 = 4y - 5 + 5$

$\frac{4y}{4} = \frac{x+5}{4}$

$y = \frac{1}{4}x + \frac{5}{4}$ $g(x)$

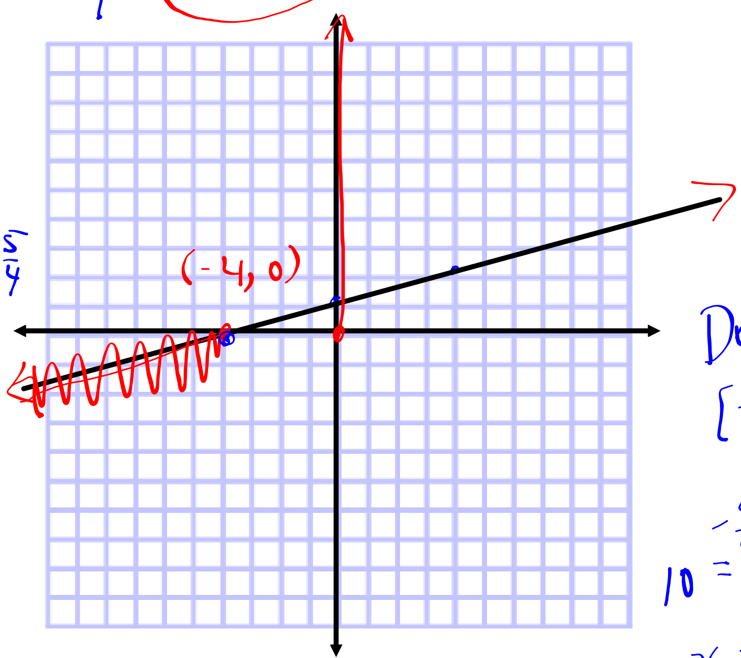
$f'(x) : \{(x,y) \in \mathbb{R} \times [0,10] \mid y = \frac{1}{4}x + \frac{5}{4}\}$

$(x, 10)$

$f'(2)$

$f'(2) = \frac{1}{4}(2) + \frac{5}{4}$

$f'(2) = 1.75$



Domain f'

$[-4, 35]$

$10 - \frac{5}{4} = \frac{1}{4}x + \frac{5}{4} - \frac{5}{4}$

$x = \frac{10 - \frac{5}{4}}{1/4}$

$x = 35$

Finding the inverse of a square root function

ex. $y = a\sqrt{b(x-h)} + k$

$f(x) = 2\sqrt{x-2} + 4$

$x - 4 = 2\sqrt{y-2} + 4 - 4$

$\frac{x-4}{2} = \frac{2\sqrt{y-2}}{2}$

$\left(\sqrt{y-2}\right)^2 = \left(\frac{x-4}{2}\right)^2$

$y - 2 + 2 = \left(\frac{x-4}{2}\right)^2 + 2$

$y = \left(\frac{x-4}{2}\right)^2 + 2$

$f^{-1}(x) = a(b(x-h))^2 + k$

$f^{-1}(x) = \left(\frac{x-4}{2}\right)^2 + 2$

$f^{-1}(x) = \left(\frac{1}{2}x - 2\right)^2 + 2$

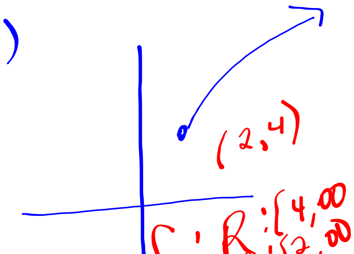
$f^{-1}(x) = \left(\frac{1}{2}(x-4)\right)^2 + 2$

Domain f^{-1}: $(4, \infty$

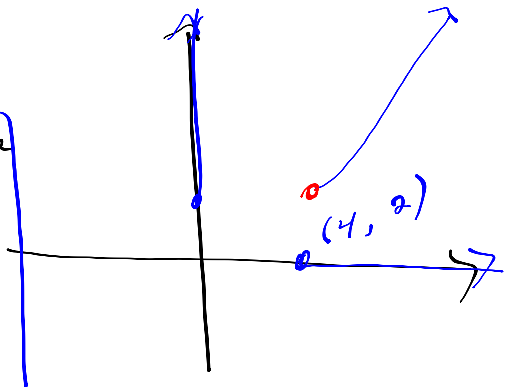
Range f^{-1}: $[2, \infty$

Note Bien:
To find the Domain and Range of the inverse, you must find the D/R of the function.

v(h,k)
2, 4



f: R: [4, ∞
D: [2, ∞



find and graph the inverse
$f(x) = -\sqrt{(x-2)} - 4$

Find the inverse of the rational function:

ex.

$$f: [-4, 8] \longrightarrow \mathbb{R}$$

$$x \longmapsto \frac{3}{x-1} + 3$$

Find $f^{-1}(x)$

Step ①: swap x, y

Step ②: Solve for y, by isolating and you'll have to cross multiple to get y out of the denominator.

$$f(x) = \frac{3}{x-1} + 3$$

$$x = \frac{3}{y-1} + 3 \quad -3 \quad -3$$

$$\frac{x-3}{1} = \frac{3}{y-1}$$

$$\frac{(x-3)(y-1)}{(x-3)} = \frac{3}{(x-3)}$$

$$y-1 = \frac{3}{(x-3)} \quad +1 \quad +1$$

$$y = \frac{3}{(x-3)} + 1$$

$$f^{-1}(x) = \frac{3}{(x-3)} + 1$$

f^{-1} D $\mathbb{R} \setminus \{3\}$
R $\mathbb{R} \setminus \{1\}$

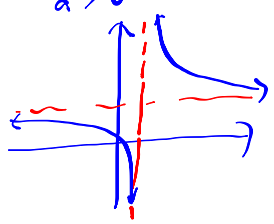
Step ③: Find the D/R of f^{-1} by finding the D/R of f and switch.....

$$f(x) = \frac{3}{x-1} + 3$$

$$y = \frac{a}{b(x-h)} + k$$

$x = h$ $\quad x = 1$
$y = k$ $\quad y = 3$

$a > 0$ QI QIII



f: D $\mathbb{R} \setminus \{1\}$
$-\infty, 1[\cup]1, \infty$

R $\mathbb{R} \setminus \{3\}$

Unit 6: Finding the Rule of a Real Function

the equation w/ its parameter

P 6.12 Finding the equation of Quadratic Function

ex. Find the equation of a quadratic function in the form $y=a(x-h)^2+k$.

$V(5,-1)$ $P(3,3)$

h k x y

$y = a(b(x-h))^2 + k$

Step ①: Assume $b=1$

Step ②: Sub in the info of the parameters you already know.

Step ③: To find missing parameter sub in a point and isolate.

$y = a(x-5)^2 - 1$

$3 = a(3-5)^2 - 1$

$3 = a(-2)^2 - 1$

$3 + 1 = a \cdot 4 - 1 + 1$

$\frac{4}{4} = \frac{a\cancel{4}}{\cancel{4}}$

$a = 1$

$$\boxed{y = (x-5)^2 - 1}$$

B
E
D
M
A
S

Find the equation of the quadratic function in the form $y = a(x - x_1)(x - x_2)$

x-ints. $\begin{cases} (x_1, 0) \\ (x_2, 0) \end{cases}$

The function has zeros $(\overset{x_1}{4}, 0)$ $(\overset{x_2}{6}, 0)$

and passes through $(\underset{x}{3}, \underset{y}{3})$

$y = a(x-4)(x-6)$

$3 = a(3-4)(3-6)$

$3 = a(-1)(-3)$

$\frac{3}{3} = \frac{a(\cancel{3})}{\cancel{3}}$

$a = 1$

$$\boxed{y = (x-4)(x-6)}$$

Find the equation of the absolute value function when given the vertex and a point

Ex: Find the absolute value equation in the form

$y = a|x-h| + k$

w/ vertex $(1, 2)$ and point $(4, 5)$

$y = a|b(x-h)| + k$

Step ①: Assume $b = 1$

$y = a|x-h| + k$

$y = a|x-1| + 2$

$5 = a|4-1| + 2$

$5 = a|3| + 2$

$5 = a \cdot 3 + 2$ -2 -2

$3 = a \cdot \cancel{3}$ $/3$ $/3$

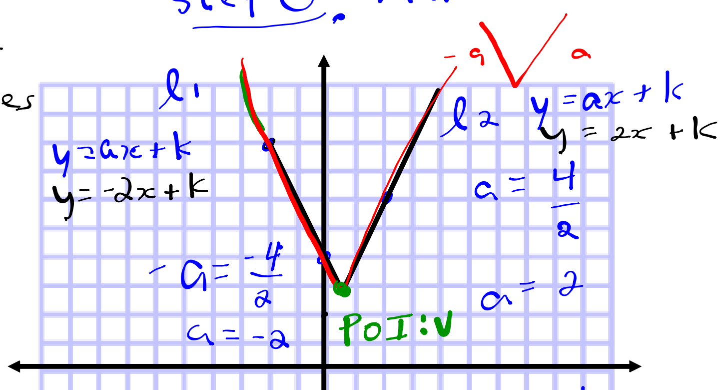
$a = 1$

$\boxed{y = |x-1| + 2}$

Find the equation of the absolute value function when given 3 points (not vertex)

e.x. Find the equation in the $y = a|x-h| + k$ if the function passes through $(-2, 8)$, $(0, 4)$, $(2, 6)$

Step ①: make a sketch.



Step 2: Find what 'a' is equal by calculating the slope of one of the lines.

Step ③: Find the point of intersection (the h, k) by constructing the linear equations of both lines and doing elimination/comparison/substitution

$y = 2|x-h| + k$

l_1: $(-2, 8)$ $(0, 4)$

$y = -2x + k$

$y = -2x + 4$

l_2: $(2, 6)$ P 6.24

$y = 2x + k$ ← y-int

$6 = 2(2) + k$

$6 = 4 + k$ (-4, -4)

$k = 2$

$y = 2x + 2$

Find POI by putting R.S. equal to each other

$-2x + 4 = 2x + 2$ ($-2x$, $-2x$, -4)

$\frac{-4x}{-4} = \frac{-2}{-4}$

$x = \frac{1}{2}$

Sub $x = \frac{1}{2}$ into $l_2: y = 2x + 2$

$y = 2\left(\frac{1}{2}\right) + 2$

$y = 3$

POI: $V\left(\frac{1}{2}, 3\right)$ (h, k) $a = 2$

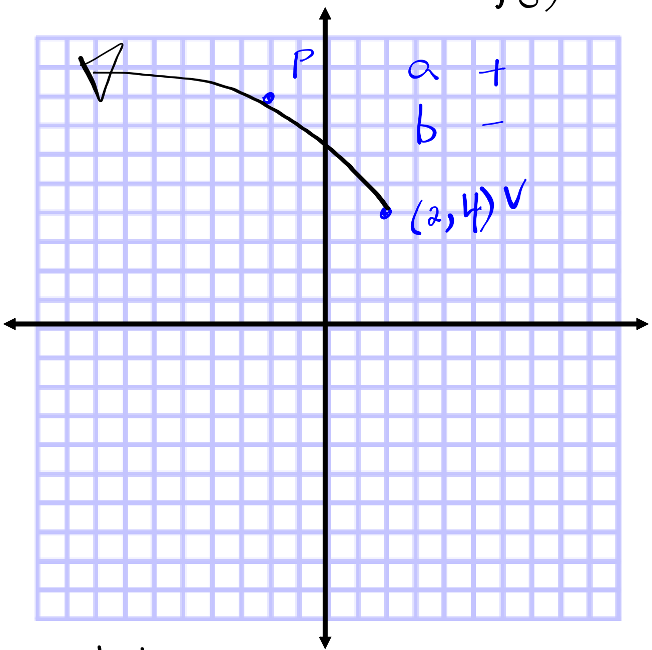
$$\boxed{y = 2\left|x - \frac{1}{2}\right| + 3}$$

Determining the Equation of a Square Root Function

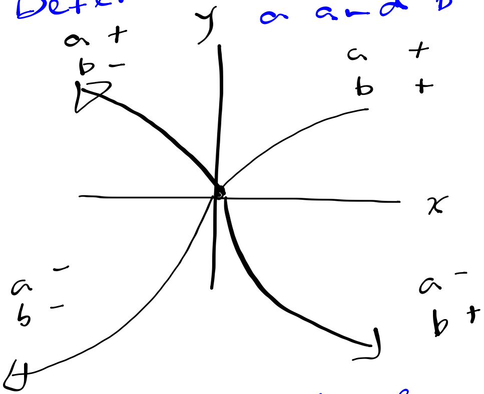
$$y = a\sqrt{b(x-h)} + k$$

ex. Find the equation of the $\sqrt{x}$ function that has vertex $\overset{h\ \ k}{(2, 4)}$

$P(-2, 8)$



Step ①: Sketch and Determine the sign of a and b



Step ②: Either assume b is equal to ± 1 and solve for 'a' by subbing in a point.

Or vise versa.

$\overset{h,\,k}{V(2,4)}$ $P(-2, 8)$

assume $b = -1$

$y = a\sqrt{b(x-h)} + k$

$y = a\sqrt{-(x-2)} + 4$

sub in $(\underset{x}{-2}, \underset{y}{8})$

$8 = a\sqrt{-(-2-2)} + 4$

$8 = a\sqrt{4} + 4$

$8 - 4 = a2 + 4 - 4$

$\frac{4}{2} = \frac{a\cancel{2}}{\cancel{2}}$ $a = 2$

$$\boxed{y = 2\sqrt{-(x-2)} + 4}$$

assume $a = 1$ find b

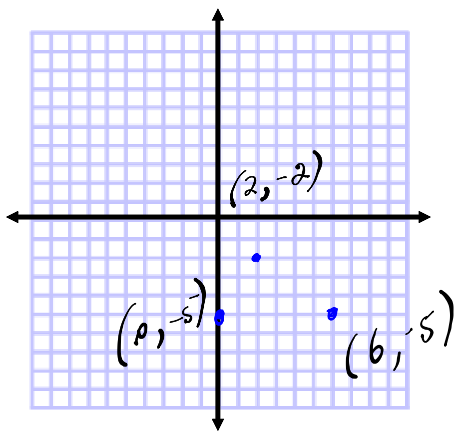
$y = \sqrt{-4(x-2)} + 4$

$y = \sqrt{4(-1)(x-2)} + 4$

$y = \sqrt{4}\sqrt{-1(x-2)} + 4$

$y = 2\sqrt{-(x-2)} + 4$

Determine the absolute value in the form
$y = a|x-h|+k$



Determine the equation of the square root function where the maximum value is -2 and the domain: $-\infty, 3]$

The function passes through P $(-4, -4)$