Unit 3: Characteristics of a Real Function

$f(x) = \{(x,y) \in]-3,3] \times \mathbb{R} \mid y = -x^2+1\}$

Recall:

Domain $]-3,3]$

Range $[-6,1]$ ← y values

axis of symmetry $x=0$

Vertex / max / min point

y-int $(0, y)$ $(3, -6)$

$f(0)$

$[\ ,\ ]$ x-value

increasing interval of the function

- where y increases as x increases
- reading left to right, the graph goes up $]-3,0[$

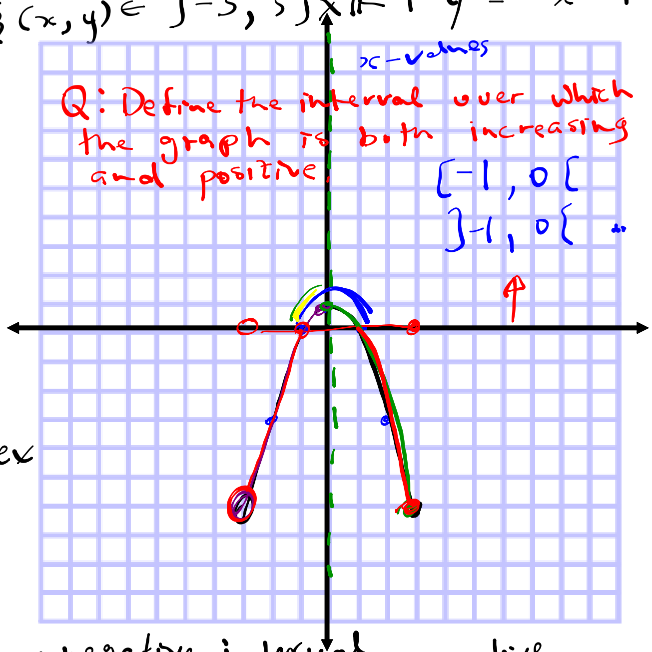
decreasing interval of the f

as x increases, y decreasing

$]0,3]$

- positive interval of the function
  - where the y-values are positive
  - where the graph is above the x-axis

$]-1,1[$



Q: Define the interval over which the graph is both increasing and positive

$[-1,0[$

$]-1,0[$

vertex

- negative interval
  - the y-values are negative
  - below the x-axis.

$]-3,-1[\cup]1,3]$

Solutions / hihi zeros / x-int(s)

$(x, 0)$ $f(x)=0$

asymptotes

slope / m / rate of change

Typical Exam Question:

if $f(x) = -|x-2| + 1$

$f(x) = a|b(x-h)| + k$

Determine $f(0)$ "find y when $x=0$"

$f(0) = -|0-2| + 1$

$f(0) = -|-2| + 1$

$f(0) = -1(2) + 1$

$f(0) = -2 + 1$

$f(0) = -1$

Determine $f(2) = 1$

$-\infty, 2[$

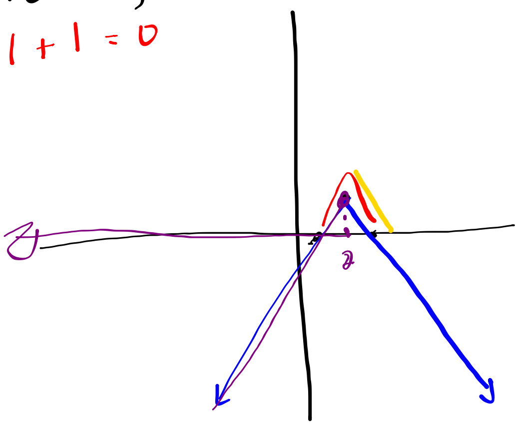
Determine the increasing interval

Determine the interval over which the function is positive and decreasing.

$]2, 3[$

2, 1

x	y
3	$-\|3-2\|+1 = -\|1\|+1 = 0$
2	1
1	0



if $f(x) = -2[-\frac{1}{4}(x+2)] - 3$

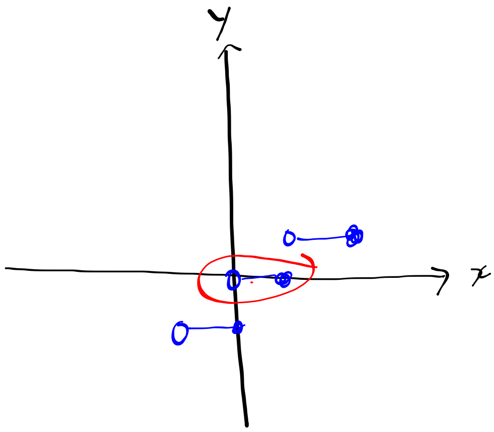
- Range
- increasing and negative
- Determine $f(3)$
- slope.

Unit 4: Solving the Equations of a Real Function (aka finding the x-ints
also known as zeros
solutions

$y=0$ find x.

- linear function
- quadratic fi
- step function (given a graph/ find x-ints)
- rational funct.
- absolute
- square root

$y=0$ find x



ex. Detern - the solutions $(x, 0)$

Step ① Put $y = 0$

Step ② Find x by isolating it by performing opposite operations to both sides.

$y = 2x - 4$

$\overset{+4}{0} = 2x - 4^{+4}$

Solve:

$\frac{4}{2} = \frac{\cancel{2}x}{\cancel{2}}$

$x = 2$ $(2, 0)$

Solve:

$4 + 2x^{-2x} = 4x^{-2x}$

$\frac{4}{2} = \frac{\cancel{2}x}{\cancel{2}}$ $x = 2$

P 4.16

Find the x-ints

$y = ax^2 + bx + c$

$y = x^2 - x - 12$

-12

1	-12
-1	12
2	-6
-2	6
-4	3

$0 = \overbrace{x^2 - x - 12}^{1\times -12}$

$0 = \underbrace{x^2 - 4x} + \underbrace{3x - 12}$

$0 = x(x-4) + 3(x-4)$

$0 = (x-4)(x+3)$

$x - 4 = 0$

$x = 4$

$x + 3 = 0$

$x = -3$

step ① put $y=0$

step ②: Find what x equal by using quadratic formula.

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$\Delta = b^2 - 4ac$

– or – factor!

if $\Delta > 0 \rightarrow$ 2 x-ints

$\Delta = 0 \rightarrow$ 1 x-int

$\Delta < 0 \rightarrow$ no x-int (don't do the quad formula)

$a = 1$

$b = -1$

$c = -12$

$\Delta = b^2 - 4ac$

$\Delta = (-1)^2 - 4(1)(-12)$

$\Delta = 49$

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{-(-1) + 7}{2(1)}$

$x = 4$

or

$x = \frac{1 - 7}{2}$

$x = -3$

Determine the solutions of an irrational function

ex $y = \frac{1}{x-2} + 4$

$0 - 4 = \frac{1}{x-2} + 4 - 4$

$\frac{-4}{1} = \frac{1}{x-2}$

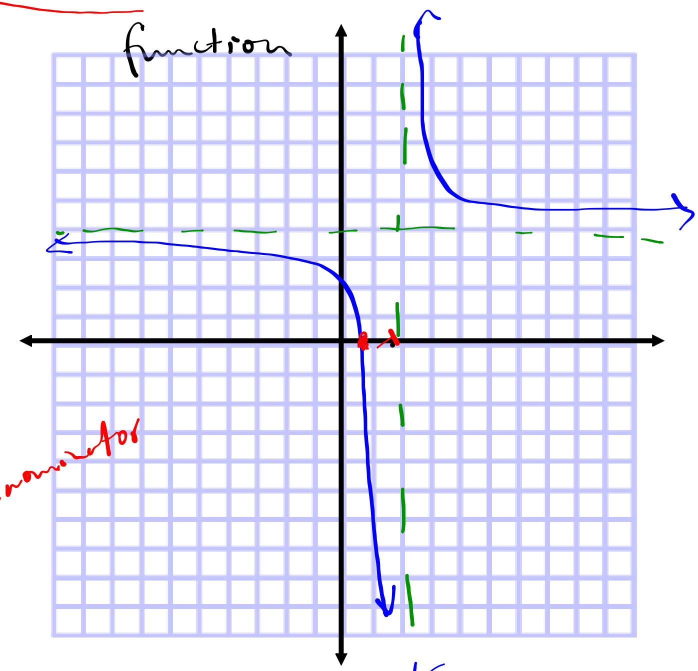
cross multiple to get x out of denominator

$-4(x-2) = 1$

$-4x + 8 - 8 = 1 - 8$

$\frac{\cancel{-4}x}{\cancel{-4}} = \frac{-7}{-4}$

$x = \frac{7}{4}$



Find x-ints

$y = \frac{-1}{x+3} + 2$

Finding the Solutions of an absolute value function

$y = a|b(x-h)|+k$

$\Delta = \frac{-k}{a}$

ex $f(x) = a|b(x-h)|+k$

$f(x) = 2|x+3| - 8$

$0^{+8} = 2|x+3| - 8^{+8}$

$\frac{8}{2} = \frac{\cancel{2}|x+3|}{\cancel{2}}$

$4 = |x+3|$

+ / −

$4 = +(x+3)$ -or- $4 = -(x+3)$

$4^{-3} = x+3^{-3}$ $\quad$ $4^{+3} = -x-3^{+3}$

$x = 1$ $\quad$ $\frac{7}{-1} = \frac{\cancel{-}x}{\cancel{-1}}$

$x = -7$

step ①: put $y = 0$

step ②: isolate the x bracket by performing opposite operations to both sides

Step ③: Consider Δ (or the # after you isolate the bracket)

$\Delta > 0 \rightarrow$ 2 solutions

$\Delta = 0 \rightarrow$ 1 solutions

$\Delta < 0 \rightarrow$ no solutions

↳ if so, stop!!

Step ④ Drop bracket by doing cases

Find the x-ints

$y = |x-1| + 1$

Solve

$3 = 3|2x-6| - 3$

Finding the zeros for a square root function

$f(x) = a\sqrt{b(x-h)} + k$

$\Delta = \frac{-k}{a}$

$\Delta \geq 0 \rightarrow$ 1 solution

$\Delta < 0 \rightarrow$ no solution (stop!!)

step ①
$y = 0$

step ②
isolate the square root sign

Step ③ isolate →
Consider Δ and continue to isolate x.

Step ④: Calculate the domain [$b(x-h) \geq 0$] and make sure your x-value belongs to the domain

$\Delta = 2$
↳ 1 solution
- all systems go

$f(x) = a\sqrt{b(x-h)} + k$

ex. $f(x) = 4\sqrt{x-2} - 8$

$0^{+8} = 4\sqrt{x-2} - 8^{+8}$

$\frac{8}{4} = \frac{\cancel{4}\sqrt{x-2}}{\cancel{4}}$

$(2)^2 = \left(\sqrt{x-2}\right)^2$

$4^{+2} = x - 2^{+2}$

$x = 6$

$\therefore x = 6 \in D$

$\therefore$ it's a good x-int.

$x - 2 \geq 0$
$x \geq 2 \leftarrow$ Domain $[2, \infty$

Find the zeros/sketch

$f(x) = -3\sqrt{6(3-x)} + 9$