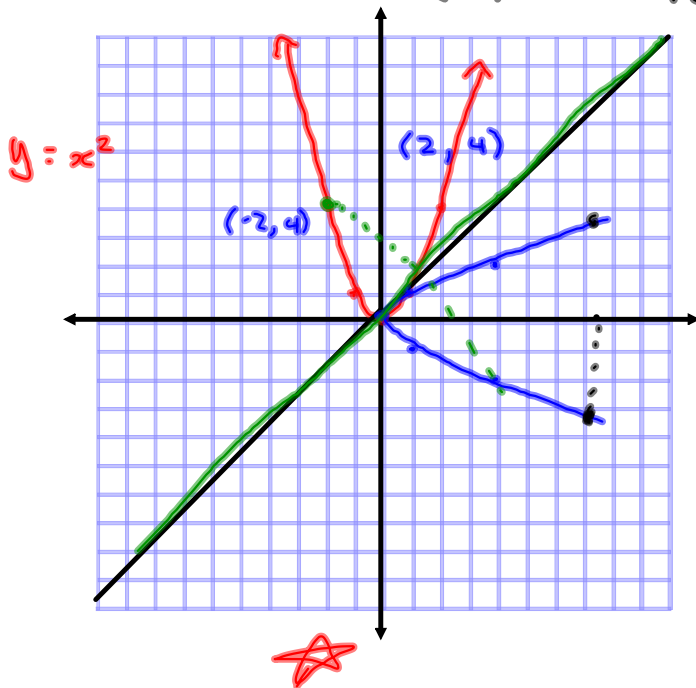


Unit 5: Inverse of a Real Function

↳ a reflection of the function around the line $y=x$



$f(x)$ - function

$f^{-1}(x)$ - inverse of a function

Invers: replace $x \leftrightarrow y$ and $y \leftrightarrow x$

Same thing for source and target set

For f^{-1}

$$\text{Dom } f^{-1} = \text{Range } f$$

$$\text{Range } f^{-1} = \text{Domain } f$$

Fx 1
p 5.2

$$f = \{ (1, 2), (2, 3), (3, 4), (4, 5) \}$$

$$\text{Dom } f = \{ 1, 2, 3, 4 \}$$

$$\text{Range } f = \{ 2, 3, 4, 5 \}$$

$$f^{-1} = \{ (2, 1), (3, 2), (4, 3), (5, 4) \}$$

$$\text{Dom } f^{-1} = \{ 2, 3, 4, 5 \} = \text{Range } f$$

$$\text{Range } f^{-1} = \{ 1, 2, 3, 4 \} = \text{Dom } f$$

Ex2
p 5.5

Find the inverse of

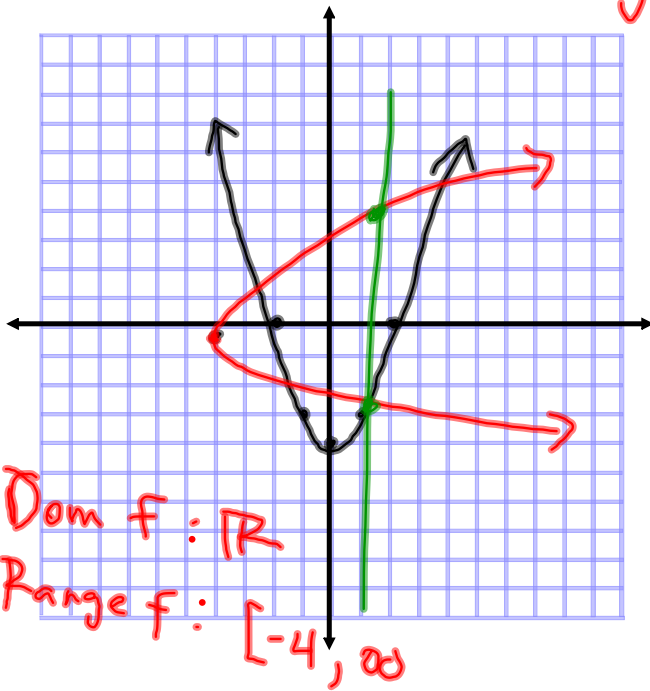
$$f(x) = x^2 - 4$$

$$y = x^2 - 4$$

$$V(0, -4)$$

$$y = a(b(x-h))^2 + k$$

$$V(h, k)$$



To find the inverse of a function algebraically

- ① Switch x and y in the equation
- ② Solve for y (isolate y)

$$\text{ex : } y = x^2 - 4$$
$$x = y^2 - 4$$

$$\sqrt{x+4} = \sqrt{y^2}$$

$$y = \pm \sqrt{x+4}$$

$$\text{Dom } f^{-1} [-4, \infty)$$

$$\text{Range } f^{-1} : \mathbb{R}$$

Find the inverse of

$$f(x) = \{ (x, y) \in [0, 10] \times \mathbb{R} \mid f(x) = 4x - 5 \}$$

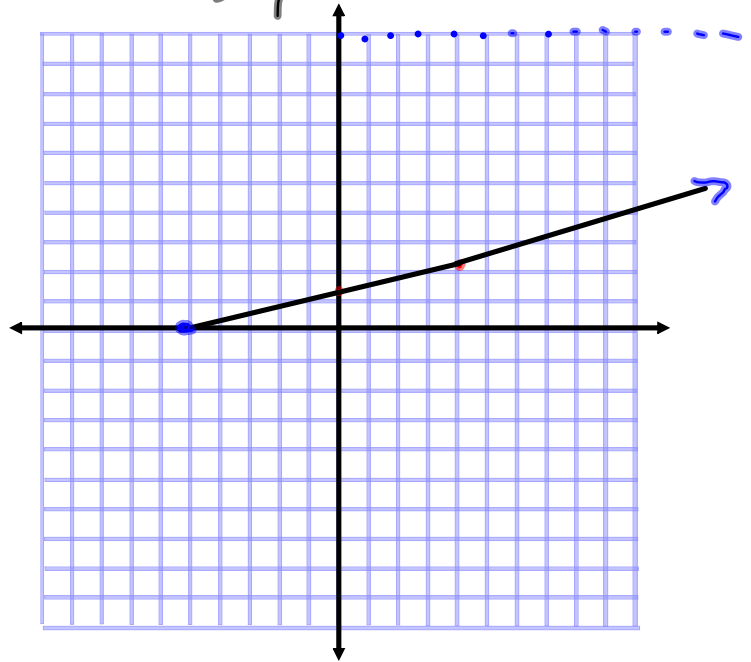
$$y = 4x - 5$$

$$x = 4y - 5$$

$$\frac{x + 5}{4} = \frac{4y}{4}$$

$$y = \frac{1}{4}x + \frac{5}{4}$$

$$f^{-1} = \{ (x, y) \in \mathbb{R} \times [0, 10] \mid f^{-1}(x) = \frac{1}{4}x + \frac{5}{4} \}$$



Ex 5

PS.29

find the inverse of

$$f(x) = 2\sqrt{x-2} + 4$$

$$f(x) = a\sqrt{b(x-h)} + k$$

$$y = 2\sqrt{x-2} + 4$$

$$x = 2\sqrt{y-2} + 4$$

$$\frac{x-4}{2} = \sqrt{y-2}$$

$$(\sqrt{y-2})^2 = \left(\frac{x-4}{2}\right)^2$$

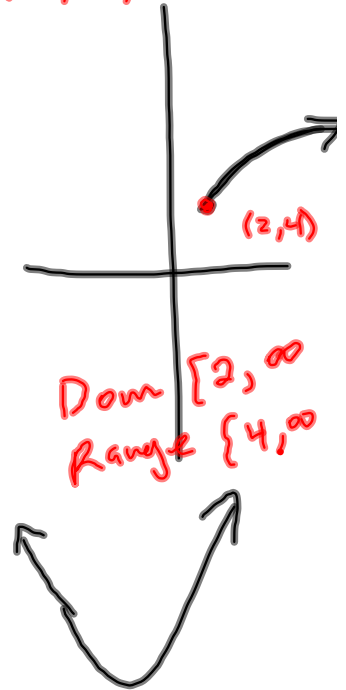
$$y = \left(\frac{x-4}{2}\right)^2 + 2$$

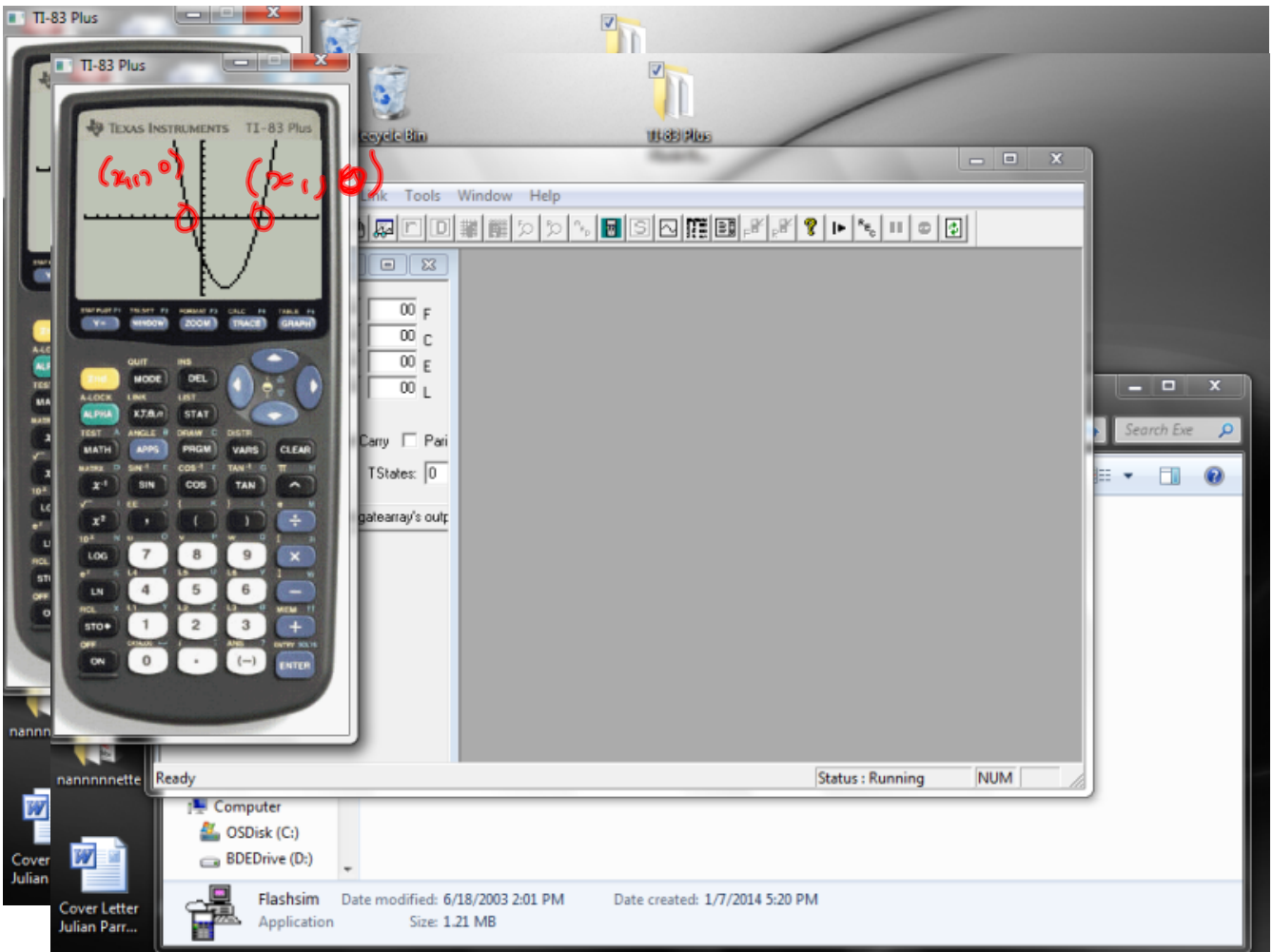
$$f^{-1}(x) = \left(\frac{x-4}{2}\right)^2 + 2$$

$$\text{Dom } f^{-1} \quad [4, \infty)$$

$$\text{Range } f^{-1} \quad [2, \infty)$$

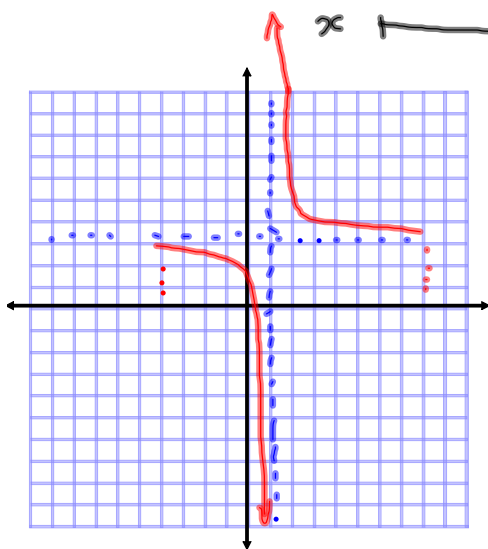
(2, 4)





Find the inverse:

$$f: [-4, 8] \rightarrow \mathbb{R}$$



$$y = \frac{a}{b(x-h)} + k$$

$x = h$
 $y = k$ } asymptote

$$y = \frac{3}{x-1} + 3$$

$$x = \frac{3}{y-1} + 3$$

$$\frac{x-3}{1} = \frac{3}{y-1}$$

$$\frac{(y-1)(x-3)}{(x-3)} = \frac{3}{x-3}$$

$$(y-1) = \frac{3}{x-3}$$

$$y = \frac{3}{x-3} + 1$$

$$x = 3$$

 $y = 1$

$$f^{-1}: \mathbb{R} \rightarrow [-4, 8]$$

$$x \longrightarrow \frac{3}{x-3} + 1$$

