Unit 6: Finding the inverse of a log function and an exp function

$y = a^x \leftrightarrow \log_a y = x$

Step ①: Find D/R of original function

$f: D \;]0,\infty = R: f^{-1}(x)$

$f: R \; \mathbb{R} = D: f^{-1}(x)$

Step ②: Switch x y.

Step ③: Solve/Isolate y by performing the opposite operation. (You'll have to convert)

Don't forget to switch D/R

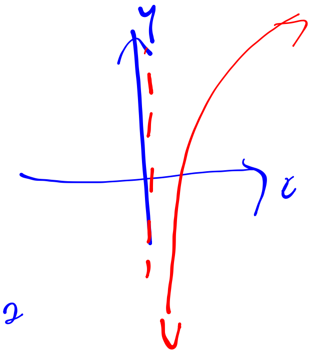
Find the inverse

$f(x) = \log_3 x$

$y = \log_3 x$

$x = \log_3 y$

$3^x = y$



$\boxed{f^{-1}(x) = 3^x \quad D: \mathbb{R} \quad R:]0,\infty}$

Find the inverse:

$f(x) = -3^x + 2$

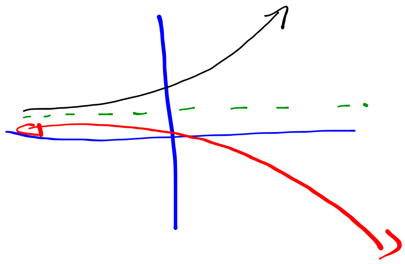
$y = -3^x + 2$

$x \; {\scriptstyle -2} = -3^y + 2 \; {\scriptstyle -2}$

$\dfrac{x-2}{-1} = \dfrac{-3^y}{-1}$

$3^y = -x + 2$

$y = \log_3(-x+2)$



* Remember the exp term needs to be isolated before conversion

$f \begin{cases} D: \mathbb{R} \\ R: -\infty, 2[\end{cases}$

$y = a^x \longleftrightarrow \log_a y = x$

$$\boxed{f^{-1}(x) = \log_3(-x+2) \quad D: -\infty, 2[\quad R: \mathbb{R}}$$

Find the inverse of:

$$f(x) = \{(x, f(x)) \in \mathbb{R} \times \mathbb{R} \mid f(x) = -\log_3 -(x + \tfrac{2}{5}) + 2\}$$

$$y = -\log_3 -(x + \tfrac{2}{5}) + 2$$

$$x \underset{-2}{=} -\log_3 -(y + \tfrac{2}{5}) + 2 \; -2$$

$$\frac{x - 2}{-1} = \frac{-\log_3 -(y + \tfrac{2}{5})}{-1}$$

$$-x + 2 = \log_3 -(y + \tfrac{2}{5})$$

$$y = $$

$$\frac{3^{(-x+2)}}{-1} = \frac{-(y + \tfrac{2}{5})}{-1}$$

$$-3^{(-x+2)} - \tfrac{2}{5} = y + \cancel{\tfrac{2}{5}} - \tfrac{2}{5}$$

$$\therefore y = -3^{(-x+2)} - \frac{2}{5}$$

$$f^{-1}(x) = \{(x, f^{-1}(x)) \in \mathbb{R} \times \mathbb{R} \mid f^{-1}(x) = -3^{(-x+2)} - \frac{2}{5}\}$$

Find the inverse of

$$y = -2^{-(x+3)} + 2$$

$$x = -2^{-(y+3)} + 2$$

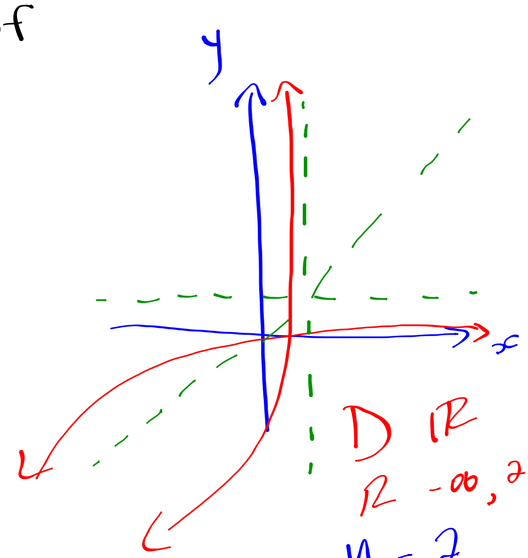
$$x - 2 = -2^{-(y+3)}$$

$$-x + 2 = 2^{-(y+3)}$$

$$\log_2 (-x+2) = -(y+3)$$

$$-\log_2(-x+2) = y + 3$$

$$y = -\log_2(-x+2) - 3$$



D ℝ

R $]-\infty, 2[$

$y = 2$

Unit 7: Laws of Logs and their application

(we use the laws to simplify expression (or to determine the value))

$y = a^x \longleftrightarrow \log_a y = x$

1st law:

$$\log_c c = 1 \qquad c^1 = c$$

e.x. simplify

$\log_2 \sqrt{4} = \log_2 2 = 1$ law 1

simplify

$\log 10$ law 1

$\log_{10} 10 = 1$

2nd law

$$\log_c 1 = 0 \longleftrightarrow c^0 = 1$$

$$\log_{\text{☺}} 1 = 0$$

$$\text{☺}^0 = 1$$

3^{rd} law

$\log_c c^n = n \longleftrightarrow c^n = c^n$

Simplify

law 3

$\log_2 4 = \log_2 2^2 = 2$

4th Law

$$\log_c M + \log_c N = \log_c MN$$

ex: Simplify

$$\log_2 16 + \log_2\left(\frac{1}{4}\right)$$

$$= 2$$

Simplify

law 4

$$\log_2(16x) = \log_2 16 + \log_2 x$$

law 3

$$= \log_2 2^4 + \log_2 x$$

$$= 4 + \log_2 x$$

5^{th} Law

$$\log_c M - \log_c N = \log_c \frac{M}{N}$$

law 5

$$\log_2 \frac{2}{x} = \log_2 2 - \log_2 x$$

law 1

$$= 1 - \log_2 x$$

6th Law

$$\log_c m^n = n \log_c m$$

ex $\log_2 2^2 \overset{6}{=} 2 \overset{\text{law 1}}{\log_2 2} = 2 \cdot (1) = 2$

Simplify

ex $\log_2 4^x \overset{\text{law 6}}{=} x \log_2 4 \overset{\text{law 3}}{=} x \log_2 2^2 = x \cdot 2 = 2x$

$\log_2 (2^2)^x = \log_2 2^{2x} = 2x$

7th Law

$$\log_{\frac{1}{c}} M = -\log_c M$$

* You don't want $\frac{1}{c}$ as a base!

If you see it, apply law 7.

Simplify / Determine the value:

$$\log_2 2 - \log_{\frac{1}{2}} 2$$

8th Law

$$\log_c m = \frac{\log_a m}{\log_a c}$$

$$\log_2 3 = \frac{\log 3}{\log 2}$$

Tips for simplifying expressions:

- if base of log is $\frac{1}{c}$, use law 7 to make it c.
- rewrite what you're taking the log of as a power w/ the same base as your log $\log_2 4 = \log_2 2^2$
- break up the logs if there's a variable (x, y, c, b...) $\log_a 16x$
- use one law at a time (correctly), be neat, and have confidence.

Given that

$\log_{10} a = 2$

$\log_{10} b = 3$

Simplify the following expression:

$$3\log_b b^2 - 5\log_a 10b + 6\log_{\frac{1}{a}} a^5 + 2\log_{\frac{1}{b}} a^3$$

$$3\cdot 2 - 5(\log_a 10 + \log_a b) - 6\log_a a^5 - 2\log_b a^3$$

$$6 - 5\log_a 10 - 5\log_a b - 6(5) - 2\cdot 3\log_b a$$

adding like terms

$$-24 - 5\log_a 10 - 5\log_a b - 6\log_b a$$

law 1

$$-24 - 5\left(\frac{\log 10}{\log a}\right) - 5\left(\frac{\log b}{\log a}\right) - 6\left(\frac{\log a}{\log b}\right)$$

$$-24 - 5\left(\frac{1}{2}\right) - 5\left(\frac{3}{2}\right) - 6\left(\frac{2}{3}\right)$$

$$= -38$$

Determine the evalue of the following expression by using the law of logs.

$\log_{\frac{1}{3}} 27$

$\log_2 16 \cdot \sqrt{32}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\log_2 2^4 \sqrt{2^5}$

$\log_2 2^4 (2^5)^{1/2}$

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\sqrt[2]{2^5} = 2^{\frac{5}{2}}$

$\log_2 2^4 2^{\frac{5}{2}}$

law ex 1

$\log_2 2^{(4 + \frac{5}{2})}$

law log 3

$\log_2 2^{(\frac{13}{2})}$

$\boxed{\frac{13}{2}}$

$\sqrt{3} = 3^{\frac{1}{2}}$

$\sqrt{4} = 4^{1/2}$