Unit 8: Solving Exp or Log Equations

(aka find the value of x)

Solve

$8 = 2^x$

$\log_2 8 = x$

$x = \frac{\log 8}{\log 2}$ 3

$x = 3$

- 3 tools

1st tool:

$y = a^x \leftrightarrow \log_a y = x$

→ generally, we use this tool when equation has 1 exp # and another #.

$x = \log_2 8$

$x = \log_2 2^3$

$x = 3$ (3)

Solve:

$3^{x+1} = 3^{4}$

put exps equal to each other, and then solve

$x + 1 = 4$ (-1 -1)

$x = 3$

tool 2 : a theorem

if $c^x = c^y$

then $x = y$

we use when there's two exp #'s and nothing else.

Solve $2^{x+1} = 2^{4x+5}$

$x + 1 = 4x + 5$ (-4x -4x)

$-3x + 1 = 5$ (-1 -1)

$\frac{-3x}{-3} = \frac{4}{-3}$

$x = -\frac{4}{3}$

Solve for x

- bring x's together & then isolate.

Check

sub $x = -\frac{4}{3}$ into equal.

$2^{x+1} = 2^{4x+5}$

$2^{4\left(-\frac{4}{3}\right)+5}$

$2^{-\frac{4}{3}+1} = 2^{-\frac{1}{3}}$

$2^{-\frac{1}{3}} = 2^{-\frac{1}{3}}$

Solve

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{64}{27}\right)^{-x-3}$$

- if you have different bases, rewrite them as powers w the same base.

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{4^3}{3^3}\right)^{-x-3}$$

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{3}{4^{-3}}\right)^{-x-3}$$ exp (3)

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\left(\frac{3}{4}\right)^{-3}\right)^{-x-3}$$ exp 7

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{3}{4}\right)^{-3(-x-3)}$$ exp (5)

Theorem 1 solve for x

$$5x - 1 = -3(-x - 3)$$

$$5x - 1 = +3x + 9$$ (-3x, -3x)

$$2x - 1 = 9$$ (+1, +1)

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

$$\left(\frac{3}{4}\right)^{5(5)-1} \stackrel{?}{=} \left(\frac{64}{27}\right)^{-5-3}$$

$$0.001 = 0.001$$

Solve

$$64^{2x-\frac{1}{2}} = \left(\frac{1}{16}\right)^{-x-1}$$

Solve

$$2^{3x-1} = 5^{x+2}$$

tool 3: a theorem

if $a^x = b^y$

then $\log_{10} a^x = \log_{10} b^y$

generally, we use ⓦ two exp #s ⓦ different bases and nothing else.

$$\log_{10} 2^{3x-1} = \log_{10} 5^{x+2}$$

$$(3x-1)\log 2 = (x+2)\log 5 \quad (6)$$

$$3x\log 2 - \log 2 = x\log 5 + 2\log 5 \quad (-x\log 5)$$

solve

$$3x\log 2 - x\log 5 - \log 2 = 2\log 5 \quad (+\log 2)$$

$$3x\log 2 - x\log 5 = 2\log 5 + \log 2$$

Factor out a gcf

① put gcf in front of bracket

② Divide terms by gcf

③ Put answer in brackets

$$\frac{x(3\log 2 - \log 5)}{3\log 2 - \log 5} = \frac{(2\log 5 + \log 2)}{(3\log 2 - \log 5)}$$

$$x = 8.32$$

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2a) – c).

5 mins

Solve $7^{4x+2} = \left(\frac{1}{343}\right)^{x-2}$ T2

Solve $\log_4(x-2) + \log_4 2x = 2$ TI

$\log_a y = x$

Solve $\log(5-x) = \log(x-3)$ TI

why not T3?

Solving

i. Figure out which solving tool to use

ii Rewrite the equations using laws of logs or of exps.

- use T2 if two exp # w same base
- use T1 if one exp #
- use T1 if more than one log and a # (cuz you can add the logs)
- use T1 when the unknown is what you're taking the log of.

(4)

$$\log_4(x-2) + \log_4 2x = 2$$

$$\log_4(2x(x-2)) = 2$$

$$4^2 = 2x^2 - 4x$$

$$16 = 2x^2 - 4x$$

$$0 = 2x^2 - 4x - 16$$

$$0 = 2(x^2 - 2x - 8)$$

$$0 = 2(x^2 - 4x + 2x - 8)$$

$$0 = 2(x(x-4) + 2(x-4))$$

$$0 = 2(x-4)(x+2)$$

$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 4 \qquad x = -2$$

$x =$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

factor trinomial

i. List factors of a·c

-8
-8 1
-4 2
-2 4
-1 8

ii Pick factors that add up to b.

iii Rewrite b term w factors.

factor out gcf

i. Identify gcf & in front of ( )

ii Divide each term by gcf

iii answer in brackets

The United Nations (UN) is an organisation ....

The UN

p 8.27 e) Solve

$\log_3(25y^2-4) - \log_3(3y+1) - \log_3(5y-2) = 1$

$\log_a y = x$

$\log_3 \frac{(25y^2-4)}{(3y+1)} - \log_3(5y-2) = 1$

$\log_3 \frac{(25y^2-4)}{(3y+1)} \div \frac{(5y-2)}{1} = 1$

simplify 1st by factoring

$\log_3 \frac{(25y^2-4)}{(3y+1)} \times \frac{1}{(5y-2)} = 1$

$-x^2$

$\sqrt{25y^2}$ $\sqrt{4}$

$5y$ 2

$(5y-2)(5y+2)$

$\log_3 \frac{(25y^2-4)}{(3y+1)(5y-2)} = 1$

$\log_3 \frac{(5y-2)(5y+2)}{(3y+1)(5y-2)} = 1$

convert to exp.

$\log_3 \left(\frac{5y+2}{3y+1}\right) = 1$

$3^1 = \frac{5y+2}{3y+1}$

$3(3y+1) = 5y+2$

$9y + 3 = 5y + 2$ $-5y$ -3

$4y = -1$

$\frac{4y}{4} = \frac{-1}{4}$

$y = -\frac{1}{4}$