

given that:

$$\log a = 2$$

$$\log b = 3$$

Simplify the following expression:

Evaluate

$$3 \log_b b^2 - 5 \log_a (10b) + 6 \log_{\frac{1}{a}} a^5 + 2 \log_{\frac{1}{b}} a^3$$

$$3 \cdot 2 - 5 [\log_a 10 + \log_a b] - 6 \log_a a^5 - 2 \log_b a^3$$

$$6 - 5 \frac{\log 10}{\log a} - 5 \frac{\log b}{\log a} - 6(5) - 2 \cdot 3 \log_b a^3$$

$$6 - 5 \left(\frac{1}{2} \right) - 5 \left(\frac{3}{2} \right) - 30 - 6 \frac{\log a^3}{\log b}$$

$$6 - \frac{5}{2} - \frac{15}{2} - 30 - 6 \left(\frac{2}{3} \right)$$

$$\boxed{-38}$$

strategies for simplifying

• one law at a time

• use law 7 to get rid of fraction bases

• use law 4/5 to break up logs w variables

• use law 8 to get variables out of base.

log
of b)
c)

$$\log_{\frac{1}{5}} \frac{\sqrt[2]{25}}{\sqrt[2]{5}}$$

$$\log_{\frac{1}{5}} \frac{25^{\frac{1}{2}}}{5^{\frac{1}{2}}}$$

$$\log_{\frac{1}{5}} \frac{(5^2)^{\frac{1}{2}}}{5^{\frac{1}{2}}}$$

$$\log_{\frac{1}{5}} \frac{5}{5^{\frac{1}{2}}}$$

$$\log_{\frac{1}{5}} 5^{1-\frac{1}{2}}$$

exp ⑤

exp ②

always convert radicals to exp #'s

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\log_{\frac{1}{5}} 5^{\frac{1}{2}}$$

$$- \log_5 5^{\frac{1}{2}}$$

$$- \frac{1}{2}$$

⑦

③

Evaluate

$$\log_{\frac{1}{2}} 8^{\frac{1}{3}} - \log_2 16^3$$

$$= \frac{\log_2 8^{\frac{1}{3}}}{-1} - \frac{\log_2 16^3}{-1}$$

Before using law 5

$$\log_c (m) - \log_c N = \log_c \left(\frac{m}{N} \right)$$

problem!
must factor out negative

$$= -(\log_2 8^{\frac{1}{3}} + \log_2 16^3)$$

$$= -(\log_2 2^{\frac{1}{3} \cdot 16^3})$$

$$= -(\log_2 (2^3)^{\frac{1}{3}} \cdot (2^4)^3)$$

exp 5

$$= -(\log_2 2^1 \cdot 2^{12})$$

exp 1

$$= -\log_2 2^{13}$$

log 3

$$= -13$$

Rewrite the following as one single log

$$3 \log_2 9 - \frac{1}{2} \log_2 27 \quad (4)$$

$$\log_2 9^3 - \log_2 27^{\frac{1}{2}}$$

$$\log_2 \frac{9^3}{27^{\frac{1}{2}}} \quad (5)$$

$$a \log_c m - b \log_c N = \log_c \frac{m^a}{N^b}$$

law (1)

$$\log_c m^n = n \log_c m$$

$$\log_2 \frac{(3^2)^3}{(3^3)^{\frac{1}{2}}}$$

$$\log_2 \frac{3^6}{3^{\frac{3}{2}}}$$

$2 \times 6 = 12$

$$\log_2 3^{\frac{9}{2}}$$

exp (2)

$$\log_2 3^{\frac{9}{2}}$$

- 3 b) 7.25
- 2 c) 7.24

Unit 8: Solving Exp or Log Equations

aka find value of x $x = \underline{\quad}$

Solve:

3 tools:

$$8 = 2^x$$

$$x = 3$$

$$\log_2 8 = x$$

$$x = \frac{\log 8}{\log 2}$$

$$x = 3$$

1st tool:

$$y = a^x \rightarrow \log_a y = x$$

Use when there's
one exp # or one
log w/ another #

Solve:

$$3^{x+1} = 3^4$$

$$x+1 = 4-1$$

$$x = 3$$

Solve

$$2^{x+1} = 2^{4x+5}$$

$$x+1 = 4x+5$$

$$-4 = \frac{3x}{3}$$

$$x = \frac{-4}{3}$$

$$x = 3$$

tool 2: a theorem
if $c^x = c^y$
then $x = y$

put exponents equal to each other and solve

$$f(x) = \left(\frac{3}{4}\right)^{5x-1} \quad \text{solve}$$

$$h(x) = \left(\frac{64}{27}\right)^{-x-3}$$

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{64}{27}\right)^{-x-3}$$

• 2 exp #'s,
but different
bases.

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{4^3}{3^3}\right)^{-x-3}$$

• try to rewrite
#'s as powers
w/ same base

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\left(\frac{4}{3}\right)^3\right)^{-x-3}$$

law 3 of exp $-n$

$$\left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$$

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\left(\frac{3}{4}\right)^{-3}\right)^{-x-3} \quad \text{exp } \textcircled{3}$$

$$\left(\frac{3}{4}\right)^{5x-1} = \left(\frac{3}{4}\right)^{3x+9} \quad \text{exp } 5$$

→ tool 2

Solve .

$$2^{3x-1} = 5^{x+2}$$

$$\log 2^{3x-1} = \log 5^{x+2}$$

$$(3x-1)\log 2 = (x+2)\log 5$$

$$3x\log 2 - \log 2 = x\log 5 + 2\log 5 + \log 2$$

$$\frac{3x\log 2}{x} - \frac{x\log 5}{x} = 2\log 5 + \log 2$$

$$\frac{x(3\log 2 - \log 5)}{(3\log 2 - \log 5)} = \frac{(2\log 5 + \log 2)}{(3\log 2 - \log 5)}$$

$$x = 8.32$$

tool 3: a theorem
if $a^x = b^y$
then $\log_{10} a^x = \log_{10} b^y$

(6)

Solve for x ,
but bring x 's
together

factor out x !

Solve: $7^{4x+2} = \left(\frac{1}{343}\right)^{x-2}$

Solve: $\log_4(x-2) + \log_4 2x = 2$

Solve: $\log_{\frac{1}{2}}(4x^2 - 8x + 3) - 3 = \log_{\frac{1}{2}}(2x^2 + 2x - 4)$

Solving:

① Figure out which tool

② Rewrite the equation before using tool, by using the law of logs.

$$4x + 2 = -3x + 6$$

$$\frac{7x}{7} = \frac{4}{7}$$

$$x = \frac{4}{7}$$

Solve!

$$\textcircled{4} \\ \log_4(x-2) + \log_4 2x = 2$$

$$\log_4(x-2)(2x) = 2$$

$$\log_4(2x^2 - 4x) = 2$$

$$4 = 2x^2 - 4x$$

$$16 = 2x^2 - 4x$$

$$0 = 2x^2 - 4x - 16$$

$ax^2 + bx + c$

$y = a^x \rightarrow \log_a y = x$
convert!

$x =$

2nd degree?
use quad formula.

$$a = 2 \\ b = -4 \\ c = -16$$

$$D = b^2 - 4ac \\ x = \frac{-b \pm \sqrt{D}}{2a}$$