Unit 7: Laws of Logs and Their Application

(to simplify an expression or to evaluate)

1st law

$$\log_c c = 1$$

ex. simply

$\log_2 \sqrt{4}$

$\log_2 2$

1

ex. simplify

$\log_{\text{☺}} \text{☺} = 1$

ex. simplify

$\log_{10} 10 = 1$

2nd law

$\log_c 1 = 0$

$y = a^x \rightarrow \log_a y = x$

$c^0 = 1$

ex. simply

$\log_{\square} 1$

0

e.x. $\log_{10} 1 = 0$

3rd law

$\log_c c^n = n$

$c^n = c^n$

S.I. rewrite #'s as exponential #'s w/ same base as that of log.

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simplify

$\log_2 x^2$

can't simplify yet

simplify

$\log_3 3^x$

x

simplify

$\log_2 4$

$\log_2 2^2 = 2$

4th Law

$$\log_c M + \log_c N = \log_c M \cdot N$$

Simplify

$\log_2 16 + \log_2 \frac{1}{4}$

$\log_2 16\left(\frac{1}{4}\right)$ (4)

$\log_2 4$

$\log_2 2^2$ (3) $= 2$

simplify

$\log_3 \frac{1}{3} + \log_3 18$

$\log_3 \left(\frac{1}{3}\right)(18)$ (4)

$\log_3 6$

Simplify

$\log_2 16x$

$\log_2 2^4 x$ (4)

(3)

$\log_2 2^4 + \log_2 x$

$4 + \log_2 x$

Simplify

$\log_3 3x$

(1)

$\log_3 3 + \log_3 x$ (4)

$1 + \log_3 x$

Sit: Break up what you're taking the log of, if there's a variable

5th Law

$M = 2$
$N = x$
$c = 2$

$$\log_c M - \log_c N = \log_c \frac{M}{N}$$

Simplify

$\log_2 \frac{2}{x}$ ⑤

① $\log_2 2 - \log_2 x$

$1 - \log_2 x$

simplify

$\log_2 (x^2 - 4) - \log_2 (x-2)$

$\log_2 \frac{(x^2-4)}{(x-2)}$

now factor!

adv 4 factoring

- gcf
- trinomial
- difference of squares.

$x^2 - 4$

$\sqrt{x^2} = x$
$\sqrt{4} = 2$

$(x+2)(x-2)$

$\log_2 \frac{(x+2)(x-2)}{(x-2)}$

$\log_2 (x+2)$

6th Law

$$\log_c m^n = n \log_c m$$

ex. $\log_2 2^2$ ⑥

① $2 \cdot \log_2 2$

$2 \cdot (1) = 2$

Simplify

$\log_2 4^x$

$\log_2 (2^2)^x$

$\log_2 2^{2x} = 2x \log_2 2$ ①

$= 2x$

7th Law

$$\log_{\frac{1}{c}} m = -\log_c m$$

STII: When you see $\frac{1}{c}$, just always apply law?

ex.

simplify

$$\log_2 2 - \log_{\frac{1}{2}} 2$$

$$\log_2 2 - (-\log_2 2)$$

$$\log_2 2 + \log_2 2$$

$$\log_2 2 \cdot 2 = \log_2 4 = \log_2 2^2 = 2$$

8th Law

$$\log_c m = \frac{\log_{10} m}{\log_{10} c}$$

Tips for simplifying expressions:

S.I. rewrite #'s as exponential #'s w/ same base as that of log.

S.II: Break up what you're taking the log of, if there's a variable

S.III: When you see $\frac{1}{c}$, just always apply law 7

S.IV: Use law 8 if helpful

→ be neat / do one law at a time / show work.

Given that

$\log a = 2$

$\log b = 3$

Simplify:

$$3\log_b b^2 - 5\log_a 10^5 + 6\log_{\frac{1}{a}} a^5 + 2\log_{\frac{1}{b}} a^3$$

$$3 \cdot 2 - 5(\log_a 10 + \log_a b) + 6(-\log_a a^5) - 2\log_b a^3$$

$$6 - 5\log_a 10 - 5\log_a b - 6\log_a a^5 - 2\log_b a^3$$

$$6 - 5\left(\frac{\log 10}{\log a}\right) - 5\left(\frac{\log b}{\log a}\right) - 6(5) - 6\left(\frac{\log a}{\log b}\right)$$

$$6 - 5\left(\frac{1}{2}\right) - 5\left(\frac{3}{2}\right) - 30 - 6\left(\frac{2}{3}\right)$$

$$6 - 30 - \frac{5}{2} - \frac{15}{2} - \frac{12}{3}$$

$$-24 - \frac{20}{2} - 4$$

$$-24 - 10 - 4$$

$$-28 - 10$$

$$-38$$

7.28

9 b) or 9 c)

5 - 10 min

if $\log_{10} x = 5$

$\log_{10} y = 2$

$$-2\log_{\frac{1}{x}} y^4 - 5\log_y (3x)^2 + 6\log_y x^3 y - 3\log_2 1$$

$$2\log_x y^4 - 10\log_y (3x) + 6(\log_y x + \log_y y)$$

$$8\log_x y - 10(\log_y 3 + \log_y x) + 6\log_y x + 6\log_y y$$

$$8\left(\frac{\log y}{\log x}\right) - 10\left(\frac{\log 3}{\log y}\right) - 10\left(\frac{\log x}{\log y}\right) + 6\left(\frac{\log x}{\log y}\right) + 6$$

$$8\left(\frac{2}{5}\right) - 10\left(\frac{\log 3}{2}\right) - 10\left(\frac{5}{2}\right) + 6\left(\frac{5}{2}\right) + 6$$

$$8\left(\frac{2}{5}\right) - 5\log 3 - 2\left(\frac{5}{2}\right) + 6$$

$$3.2 - 5\log 3 - 10 + 6$$

$$-0.8 - 5\log 3 = -3.186\ldots$$

Simplify

$3\log_2 9 - \frac{1}{2}\log_2 27$

Before using law 5, you must get rid of coefficients.

$\log_2 9^3 - \log_2 27^{1/2}$ (6)

$\log_c m - \log_c N = \log_c \frac{m}{N}$

$\log_2 \frac{9^3}{27^{1/2}}$

$\log_2 \frac{(3^2)^3}{(3^3)^{1/2}}$ exp 5

$\log_2 \frac{3^6}{3^{3/2}}$ exp 2

$\log_2 3^{6-\frac{3}{2}}$

$= \log_2 3^{\frac{9}{2}}$

$= 7.13$

Evaluate

$$\log_{\frac{1}{2}} 8^{\frac{1}{3}} - \log_2 16^3$$

$a\log_c m + b\log_c N = \log_c m^a \cdot N^b$

3 b) P 7.25

2 c) P 7.24

Simplify

$\log_2 16 \cdot \sqrt{32}$

$\log_2 2^4 \cdot \sqrt{32}$

$\log_2 2^4 \cdot 32^{\frac{1}{2}}$

$\log_2 2^4 \cdot (2^5)^{\frac{1}{2}} = \log_2 2^4 \cdot 2^{\frac{5}{2}} = \log_2 2^{2\times\frac{4}{2}+\frac{5}{2}}$

$= \log_2 2^{\frac{13}{2}}$

$= 13/2$

① ② ③ ④ ⑤ ⑥

when you see a radical, esp w another exp #, convert it to an exp #

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

HMWK

P 7.13 Ex 7.3

P 7.24 Practise Q.

(Don't use calculator Nicolas) — for the logs, tool?

not Q.4 not Q.5 not Q.6