Unit 3: Converting Exponential Expression into Logarithmic Expression

$y = a^x \longrightarrow \log_a y = x$

exponent

base of exp number

base of the log

reasons to convert
- to find the value of an exp.
- to graph log functions
- to find the value of the log (what you're taking the log of)

ex. Solve $8 = 2^x$

$2^3 = 2 \cdot 2 \cdot 2$

$x = 3$

$\log_2 8 = x$

$\frac{\log 8}{\log 2} = x$

$3 = x$

$\log_{10}$

$\log_c m = \frac{\log m}{\log c}$

Solve $243 = 3^x$

$\log_3 243 = x$

$x = \frac{\log 243}{\log 3}$

$x = 5$

$y = 3^{x-1} + 4$

$y = a c^{b(x+h)} + k$

Solve for x

$13 = 3^{x-1} + 4$ (-4 on both sides)

$9 = 3^{x-1}$

$\log_3 9 = x - 1$ (+1 on both sides)

$\log_3 9 + 1 = x$

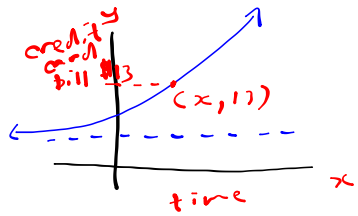
$x = \frac{\log 9}{\log 3} + 1$

$x = 3$

$c = 3$ $h = 1$

$a = 1$ $k = 4$

$b = 1$



$y = a^x \longleftrightarrow \log_a y = x$

To solve

step ① perform opposite operation and isolate the exp number

step ② convert to log and continue to isolate

BEDMAS

$3^{3-1} + 4 = 13$ ✓

$9 + 4 = 13$

i. Converting to solve for x in exp

Solve for x

$-2^{-x+1} - 3 = -5$

the exp number must be isolated w/ no coefficients.

$0 = -2^{-x+3} + 4$

find the x-int

$y = 3^{-(x-2)} - 9$

Find the positive interval

of $y = -2^{-x+3} + 4$

$y = a\,c^{b(x-h)} + k$

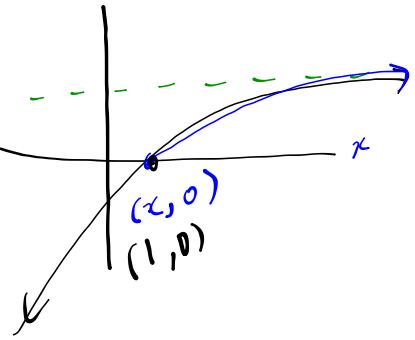
$y = -2^{-(x-3)} + 4$

$a = -1$ $b = -1$ $c = 2$

$h = 3$

$k = 4$

pos. y: $]1, \infty$



ii Converting to solve for x in log

Solve

$-\log_2(x+1) + 7 = -9$ -7

$\frac{-\log_2(x+1)}{-1} = \frac{-16}{-1}$

$\log_2(x+1) = 16$

$2^{16} = x + 1$ -1 -1

$x = 2^{16} - 1$

$x = 65535$

To solve

step ①: Isolate the log term.

step ②: get rid of log by converting to exp form

$y = a^x \leftrightarrow \log_a(y) = x$

step ③: continue to isolate for x

check

$-\log_2(65535+1) + 7 = -9$

$-\left[\frac{\log 65536}{\log 2}\right] + 7 = -9$

$-9 = -9$

Solve

$-2\log_2(x+3) + 4 = -12$

iii Converting to find the inverse or graphing purposes

ex. convert

$y = \log_2(x+1) + 7$

$y - 7 = \log_2(x+1)$

$x + 1 = 2^{y-7} - 1$

$x = 2^{y-7} - 1$

inverse

$y = 2^{x-7} - 1$

$y = a^x \longleftrightarrow \log_a y = x$

to convert

step ① first isolate log term.

step ②: convert

step ③: isolate x.

Convert to exponential expression

$y = a^x \iff \log_a y = x$

$-\log_2 x + 1 = y$

$-4 - \log_x a = y$

$\log_{\square}(ax) = -y + 1$

Convert to log exp

$-2^x - 3 = y$

$y - 4$

$x - 2 = -3$

$2.13 + 1$

$A(t) = A_0\left(1 + \frac{r}{k}\right)^{kt}$

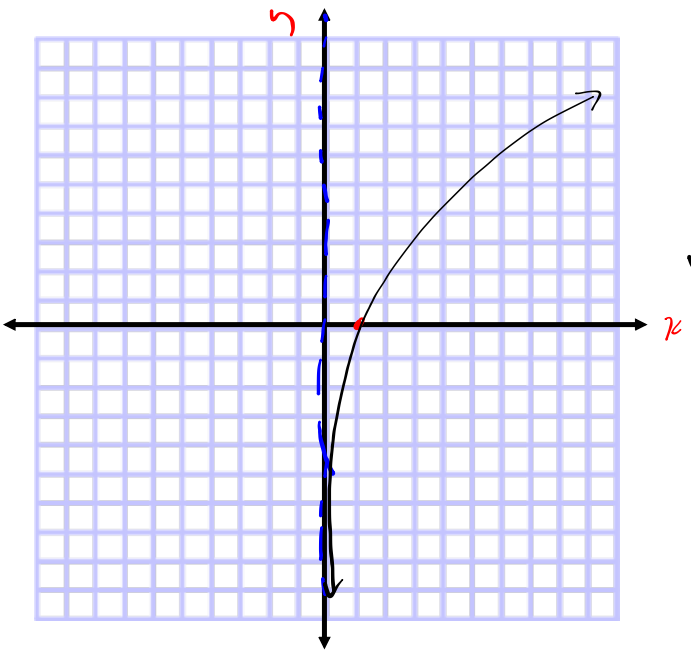
How long will it take for the computers to have a value of $1000

$1000 =$

Unit 4: Graphing Log Function

$f(x) = a \log_c(b(x-h)) + k$, asymptotes $x = h$

$x = 0$



initially

$a = 1$ $h = 0$ $c = 2$

$b = 1$ $k = 0$

$y = \log_2 x$

Recall:

a - up/down scale

b - left/right scale

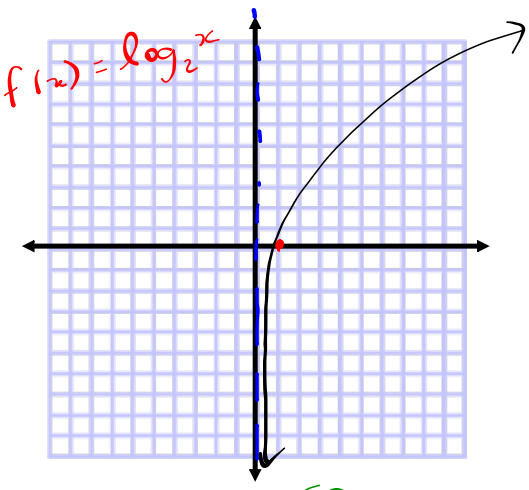
h - horizontal translation

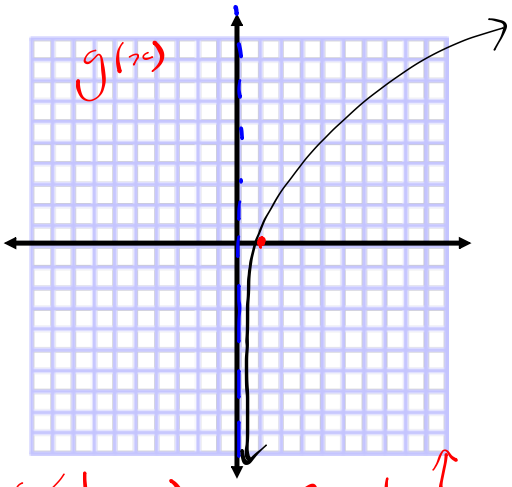
k - vertical translation

c does a's job

$c > 1$ normal

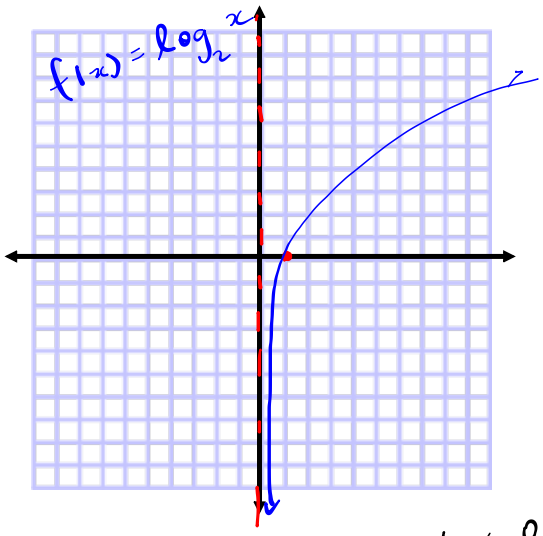
$0 < c < 1$ up/flip down





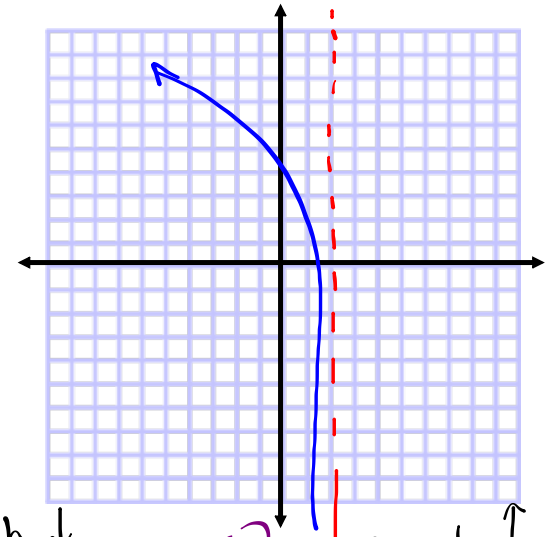
a) $a < 0$, $0 < c \leq 1$ c) $a < 0$ $h \uparrow$

b) $k \downarrow$, $a > 0$ d) $a < 0$ $b < 0$



a) $b < 0$ h ↓
$0 < c < 1$

b) $b < 0$



c) $b < 0$ h ↑

d) $b < 0$ k ↑

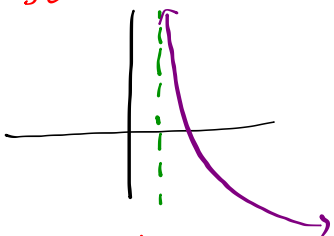
graph

$f(x) = -2\log_3 2(x-2) + 1$

$f(x) = a\log_c b(x-h) + k$

$a = -2$ $h = 2$ $c = 3$

$b = 2$ $k = 1$



To graph:

step ① Consider the parameter/sketch.

step ②: Before TOV, convert to exp expression and isolate x. (solve for x)

$y = -2\log_3 2(x-2) + 1$

$y - 1 = -2\log_3 2(x-2)$

$\frac{y-1}{-2} = \log_3 2(x-2)$

$\frac{\frac{y-1}{-2}}{2} = x - 2$

$\frac{\frac{y-1}{-2}}{2} + 2 = x$

$x = \frac{\frac{y-1}{-2}}{2} + 2$

step ③ Construct TOV pick y values

x	y
	k+2
	k+1
	k
	k-1
	k-2

x	y	
3.5	-1	
2.87	0	K
2.5	1	S
2.29	2	L
2.17	3	D

when $y = -1$

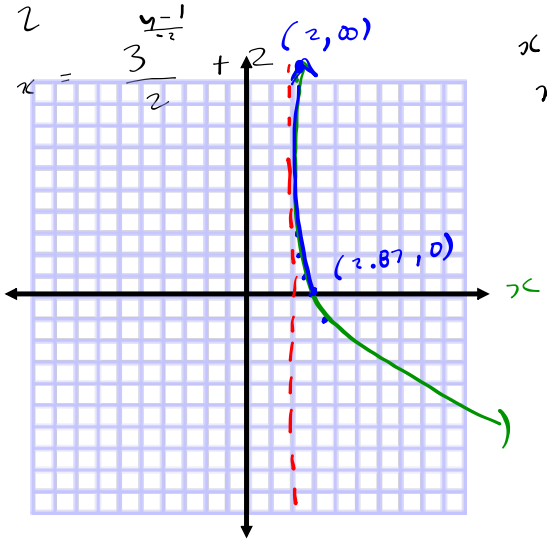
$\frac{-1-1}{-2}$

$x = \frac{3}{2} + 2$

$x = \frac{3}{2} + 2$

$x = 3.5$

step ④ Draw asymptotes, plot points, draw curve.



Domain $]2, \infty$

Range $\mathbb{R}$

pos. tive $]2, 2.87[$

graph

$f(x) = \log_{\frac{1}{2}}(-3x + 2) + 5$

- state D and R
- and state positive interval!

homework:

p 4.47 - 4.49
#1 / 2
p 1.40 - 1.44
#2 - 4
p 2.18 #4 / 5

p 3.9
#1 3
p 3.13
#2

if
$f(x) = -\log_2(-2x + 10) - 4$

- determine whether f(x) is strictly increasing for $x < 5$
- T/F: the asymptote is $y = -4$
- determine the y-int
- determine the negative interval.