Unit 4 : The Sum/Difference/Product and Quotient of Radicals

- operations on radicals
- simplify radicals
- rationalize the denominator

You can only add like radicals

$1\sqrt[2]{2}$

1) $\sqrt[3]{2}$
2) $2\sqrt{3}$
3) $3\sqrt{2}$

★ like radicals must have the same root index and the same base (radicand)

★ like radicals can have different coefficients

e.x. $\sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

e.x. $\sqrt{2} - 3\sqrt{2} = -2\sqrt{2}$

$x^2 + 3x^2 = 4x^2$

e.x. $\sqrt[3]{3} + \sqrt[2]{3} =$ → can't add cuz no like radicals.

e.x. $2\sqrt{2} + 4\sqrt{2} = 6\sqrt{2}$

root index; coefficient; base

To add like radicals, add their coefficients and write the same root index and base.

$$r\sqrt[n]{a} + s\sqrt[n]{a} = (r+s)\sqrt[n]{a}$$

Add ?

$\sqrt{32} + \sqrt{2}$

* Most of the time you will have to simplify the radical before adding it → rewrite the base as a product (x) of its factors (ex 4 and 2 are factors of 8) and pick the factors that are squares. (ex 4 is a square cuz $\sqrt{4} = 2$ ← a whole #)

32

1 32

2 16 $\sqrt{16} = 4$

8 4

$\sqrt{32} + \sqrt{2}$

$\sqrt{16 \times 2} + \sqrt{2}$

Now follow law 1 of radicals to break up the radical

$\sqrt[n]{a \times b} = \sqrt[n]{a} \times \sqrt[n]{b}$

$\sqrt{16 \times 2} + \sqrt{2}$

$\sqrt{16} \times \sqrt{2} + \sqrt{2}$

$4\sqrt{2} + \sqrt{2}$

$5\sqrt{2}$

$\sqrt{32} = 4\sqrt{2}$

5.65

5.65

Evaluate

$\sqrt{7} + \sqrt{3} + \sqrt{27}$

$$\sqrt{7} + \sqrt{3} + \sqrt{27}$$

$$= \sqrt{7} + \sqrt{3} + \sqrt{3 \times 9}$$

$$= \sqrt{7} + \sqrt{3} + \sqrt{3} \times \sqrt{9}$$

$$= \sqrt{7} + \sqrt{3} + \sqrt{3} \times 3$$

$$= \sqrt{7} + \sqrt{3} + 3\sqrt{3}$$

$$= \sqrt{7} + 4\sqrt{3}$$

2 7

1 27

3 9

$\sqrt{9} = 3$

$4\sqrt{8} - 2\sqrt{32} - 3\sqrt{18}$

$4\sqrt{4\times2} - 2\sqrt{16\times2} - 3\sqrt{9\times2}$

$4\sqrt{4}\times\sqrt{2} - 2\sqrt{16}\times\sqrt{2} - 3\sqrt{9}\times\sqrt{2}$

$4\times2\sqrt{2} - 2\cdot4\sqrt{2} - 3\cdot3\sqrt{2}$

$8\sqrt{2} - 8\sqrt{2} - 9\sqrt{2}$

$-9\sqrt{2}$

8
1 8
2 4

3 2
1 3 2
2 16
4 8
1 8
2 16
2 8

Multiplying Radicals

Evaluate

- bases can be different
- multiply the coefficients together
- multiply the bases together under one root sign

$-2\sqrt{5} \times (-3\sqrt{2}) \times 4\sqrt{8}$

$-2 \times (-3) \times 4 \sqrt{5 \times 2 \times 8}$

$24\sqrt{80}$

80
1 80
2 40
4 20
5 16
8 10

$24\sqrt{16 \times 5}$

$24\sqrt{16} \times \sqrt{5}$

$= 24 \cdot 4 \times \sqrt{5}$

$= 96\sqrt{5}$

$r\sqrt{a} \times s\sqrt{b} = rs\sqrt{ab}$

p 4.17 d) $8\sqrt{2} \times 2\sqrt{8} \times 5\sqrt{3}$

4.17

c) $-4.5\sqrt{12} \times \frac{1 \cdot \sqrt{3}}{0.9}$

$-4.5\sqrt{12} \times \frac{1}{0.9}\sqrt{3}$

$-\frac{4.5}{1} \times \frac{1}{0.9}\sqrt{12 \times 3}$

$\frac{-4.5}{0.9}\sqrt{36}$

$-5\sqrt{4 \times 9}$

$-5\sqrt{4} \times \sqrt{9}$

$-5 \cdot 2 \cdot 3$

-30

36
1 36
2 18
3 12
4 9

4.11 and 4.17

$-5\sqrt{36}$

$-5 \cdot 6$

-30

Dividing Radicals

you can divide radicals w different bases

$$\frac{r\sqrt{a}}{s\sqrt{b}} = \frac{r}{s}\sqrt{\frac{a}{b}}$$

ex $\frac{3\sqrt{27}}{\sqrt{3}} = 3\sqrt{\frac{27}{3}} = 3\sqrt{9} = 3 \cdot 3 = 9$

Perform the following operation and simplify

$\frac{6\sqrt{48}}{18\sqrt{6}}$

$\frac{18\sqrt{80}}{-3\sqrt{4}}$

$\frac{\frac{1}{3}\sqrt{8}}{\frac{3}{2}\sqrt{2}}$

$$\frac{18\sqrt{80}}{-3\sqrt{4}} = \frac{18}{-3}\sqrt{\frac{80}{4}} = -6\sqrt{20}$$

$$= -6\sqrt{4 \times 5}$$

$$= -6\sqrt{4} \times \sqrt{5}$$

$$= -6 \cdot 2\sqrt{5}$$

$$= -12\sqrt{5}$$

20
1 20
2 10
4 5

$$\frac{\frac{1}{3}\sqrt{8}}{\frac{3}{2}\sqrt{2}} = \frac{1}{3} \times \frac{2}{3}\sqrt{\frac{8}{2}}$$

$$= \frac{2}{9}\sqrt{4} = \frac{2}{9} \cdot \frac{2}{1} = \frac{4}{9}$$

.

Nota Bene: you cannot have a radical in the denominator as a final answer. You must rationalize the denominator.

$$\frac{\sqrt{5}\times\sqrt{10}}{\sqrt{10}\times\sqrt{10}} = \frac{\sqrt{5\times10}}{\sqrt{10\times10}}$$

* To do so, times the denominator by itself. Make sure to do the same to the top.

$$\frac{\sqrt{50}}{\sqrt{100}} = \frac{\sqrt{50}}{10}$$

50
1 50
(2 25)

$$= \frac{\sqrt{25\times2}}{10} = \frac{\sqrt{25}\times\sqrt{2}}{10} = \frac{5\sqrt{2}}{10}$$

$$= \frac{1}{2}\sqrt{2}$$

Evaluate and rationalize the denominator if necessary:

$$\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{4\sqrt{3}}{3}$$

$$\frac{27\sqrt{3}}{3\sqrt{6}} = 9\sqrt{\frac{3}{6}}$$

$$= 9\,\frac{\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{9}{1}\,\frac{\sqrt{3\times 6}}{6}$$

$\sqrt{36} = 6$

$$= \frac{9}{6}\sqrt{18}$$

18

1 18

2 9

$$= \frac{9}{6}\sqrt{9\times 2}$$

$$= \frac{9}{6}\sqrt{9}\times\sqrt{2} = \frac{9}{6}\cdot 3\sqrt{2}$$

$$= \frac{27}{6}\sqrt{2}$$

$$= \frac{9}{2}\sqrt{2}$$

$$9\sqrt{\frac{1}{2}} = 9\,\frac{\sqrt{1}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= 9\,\frac{\sqrt{2}}{2}$$

4.24 # 15 Perform the following operations and rationalise the denominator

$$\frac{4}{\sqrt{3}} + \frac{7}{\sqrt{7}} - \frac{\sqrt{3}}{3}$$

$$\frac{4\sqrt{3}}{3} + \frac{7\sqrt{7}}{7} - \frac{1\sqrt{3}}{3}$$

tip: when adding, rationalize the denominator right away.

$$\frac{3}{3}\sqrt{3} + \sqrt{7}$$

$$\sqrt{3} + \sqrt{7}$$