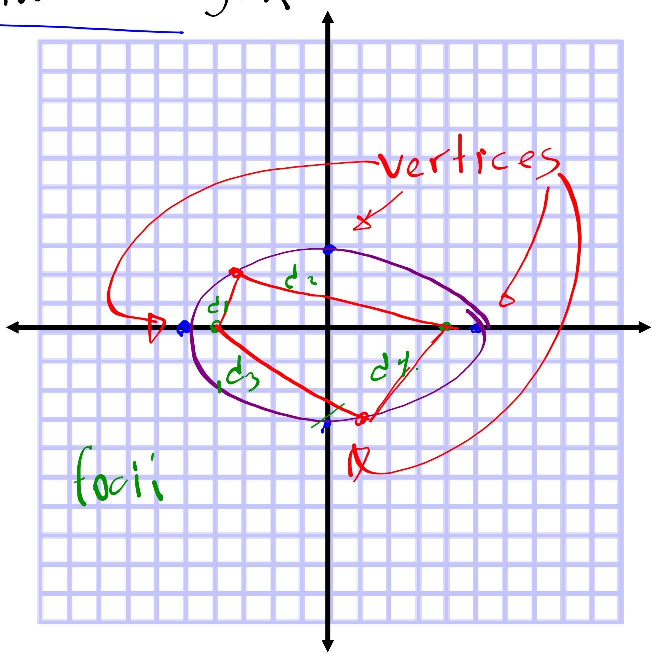
Unit 6: Graphing an Ellipse centred at the origin

Definition: An ellipse is a set of points where the sum of the distances from a point to each focus are equal.

$$d_1 + d_2 = d_3 + d_4$$



Standard Form of an Ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Vertices $(\pm a, 0)$

$(0, \pm b)$

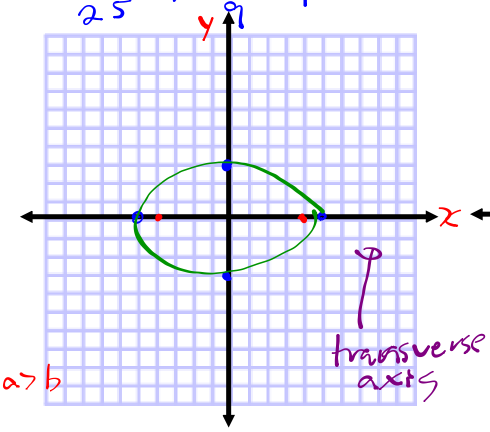
Focii $(\pm c, 0)$ if $a>b$

$(0, \pm c)$ if $b>a$

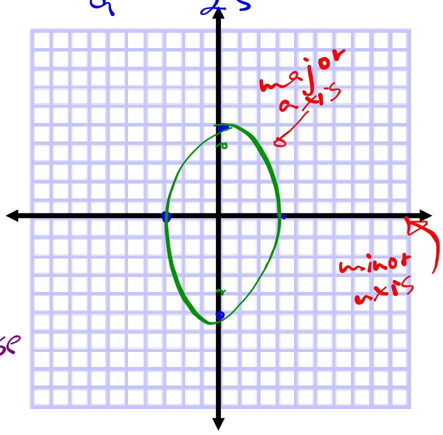
To calculate c:

$$c^2 = |a^2 - b^2|$$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$



$$\frac{x^2}{9}+\frac{y^2}{25}=1$$



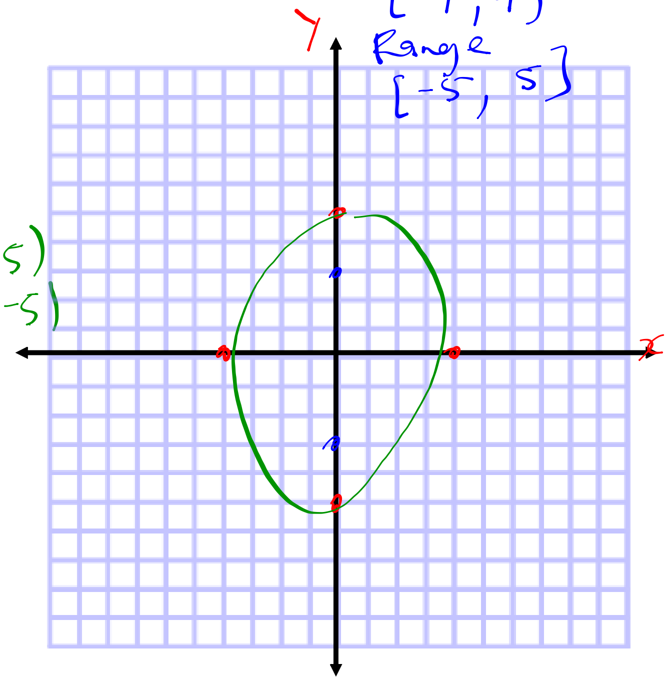
Transverse axis: / Major Axis
the line that passes through the foci

Conjugate axis / minor Axis: the perpendicular bisector to the transverse axis graph / state: (x,y) Focii / state D/R

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

Domain
$[-4, 4]$
Range
$[-5, 5]$



Find vertrces
$(\pm a, 0)$
$(0, \pm b)$

$a = \sqrt{16}$ $(4, 0)$
$a = 4$ $(-4, 0)$

$b = \sqrt{25}$ $(0, 5)$
$b = 5$ $(0, -5)$

To find focii
$c^2 = |a^2 - b^2|$
$c^2 = |4^2 - 5^2|$
$c^2 = |16 - 25|$
$c^2 = |-9|$
$\sqrt{c^2} = \sqrt{9}$
$c = 3$

Focii $(0, 3)$
$(0, -3)$

ex. graph: Focii D/R

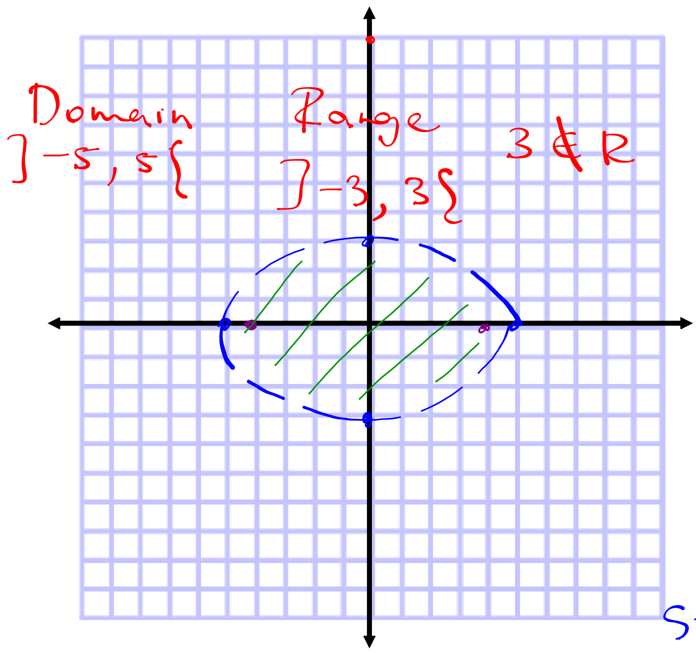
$$\frac{x^2}{25} + \frac{y^2}{49} = 1$$

graph $\frac{x^2}{16} + \frac{y^2}{4} = 1$

P6.18 graph / state focii / state Domain / Range

sub (0,10) 11.1 < 1 False

$$\frac{x^2}{25} + \frac{y^2}{9} < 1$$



Step ①: Graph corresponding equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Find vertices

$(\pm a, 0)$

$a = \sqrt{25}$ $(5,0)$

$a = \pm 5$ $(-5,0)$

$b = \sqrt{9}$ $(0,3)$

$b = 3$ $(0,-3)$

Focii $(\pm c, 0)$

$c^2 = |a^2 - b^2|$

$c^2 = |25 - 9|$

$c = 4$

Step ②: to know where to shade, sub in test point (0,0) into original inequality.

$$\frac{x^2}{25} + \frac{y^2}{9} < 1 \quad \text{sub } (0,0)$$

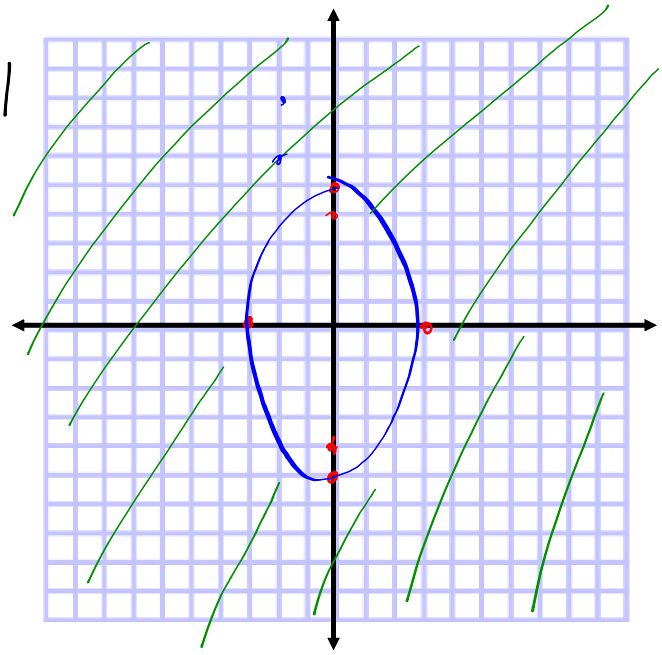
$$\frac{0^2}{25} + 0 < 1$$

$$0 < 1$$

True! so shade where (0,0) is.

graph state (x, y) foci / Domain Range

$$\frac{x^2}{9} + \frac{y^2}{25} \geq 1$$



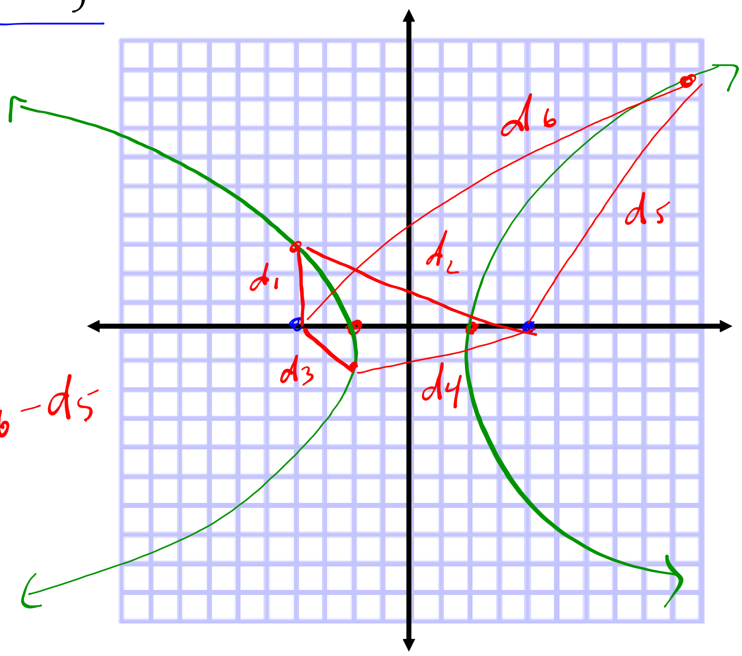
Domain $\mathbb{R}$

Range $\mathbb{R}$

Unit 7: Graphing a Hyperbola Centred at the origin

Definition: A set of of points where the differences of the distances from a point to each focus are equal.

$d_2 - d_1 = d_4 - d_3 = d_6 - d_5$



Standard form of a hyperbola facing left/right

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices $(\pm a, 0)$

Asymptotes $y = \pm \frac{b}{a} x$

Foci $(\pm c, 0)$

ex. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

To graph:

Vertices $(2, 0)$ $(-2, 0)$

$c^2 = a^2 + b^2$

$c^2 = 2^2 + 3^2$

$c^2 = 4 + 9$

$\sqrt{c^2} = \sqrt{13}$

$c = 3.61$

Foci: $(3.61, 0)$ $(-3.61, 0)$

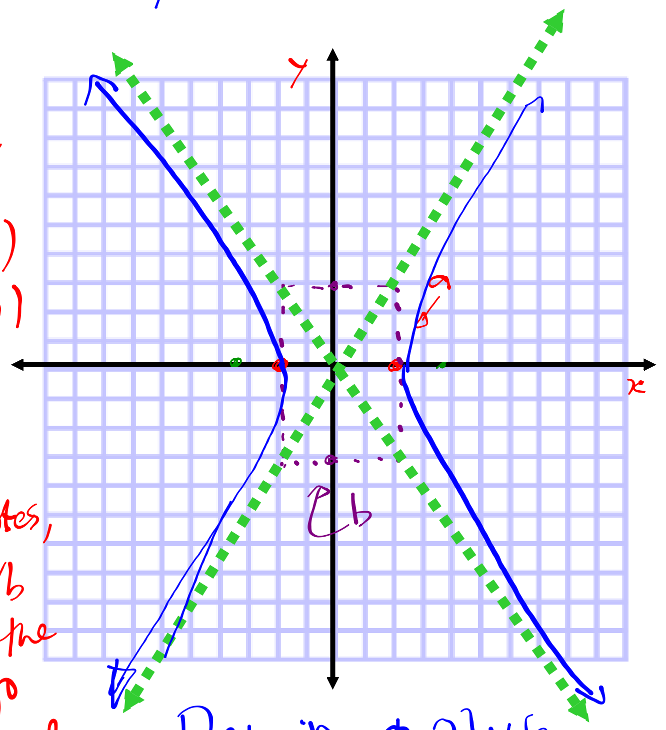
Asymptotes

$y = mx + b$

$y = \frac{3}{2} x$

$y = \frac{-3}{2} x$

For the asymptotes, draw your a/b rectangle, and the asymptotes go through the diagonal.



Domain $-\infty, 2] \cup [2, \infty$

Range $\mathbb{R}$

graph

$$\frac{x^2}{16} - \frac{y^2}{25} = 1$$

state D / R

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

graph:

V $(\pm 4, 0)$ $(\pm a, 0)$

F $(\pm 6.48, 0)$ $(\pm c, 0)$

A: $y = \pm \frac{b}{a} x$

$y = \pm \frac{5}{4} x$

$a = 4$

$b = 5$

$c^2 = a^2 + b^2$

$c^2 = 16 + 25$

$c = \sqrt{41}$

$c = 6.48$

To graph a line:

$y = mx + b$

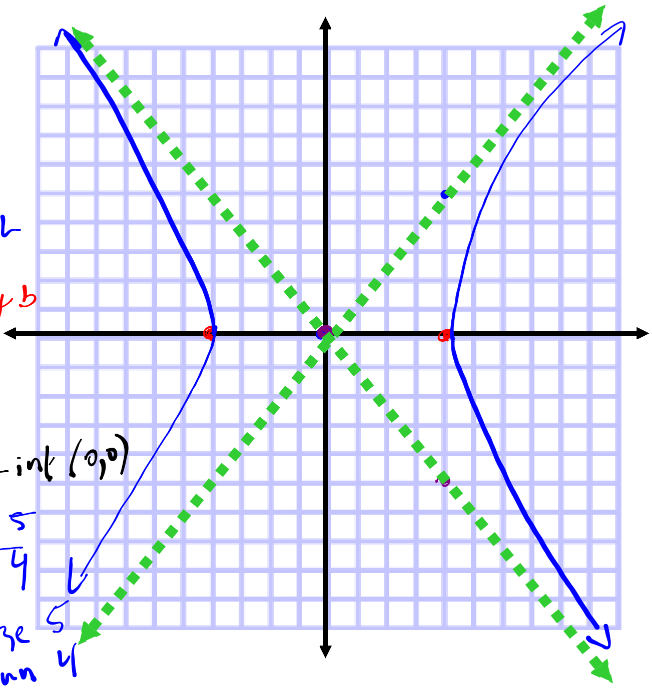
$y = \frac{5}{4} x$

plot 'b' → y-int $(0,0)$

$m = \frac{rise}{run} = \frac{5}{4}$

from b, rise 5 run 4

$y = -\frac{5}{4} x$



Standard Form of a Hyperbola

Facing up/down

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

graphing:

Vertices $(0, \pm b)$

Foci $(0, \pm c)$

$c^2 = a^2 + b^2$

Asymptotes

$y = \pm \frac{b}{a} x$

ex

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

$V(0, 3)$

$(0, -3)$

$a = 4$

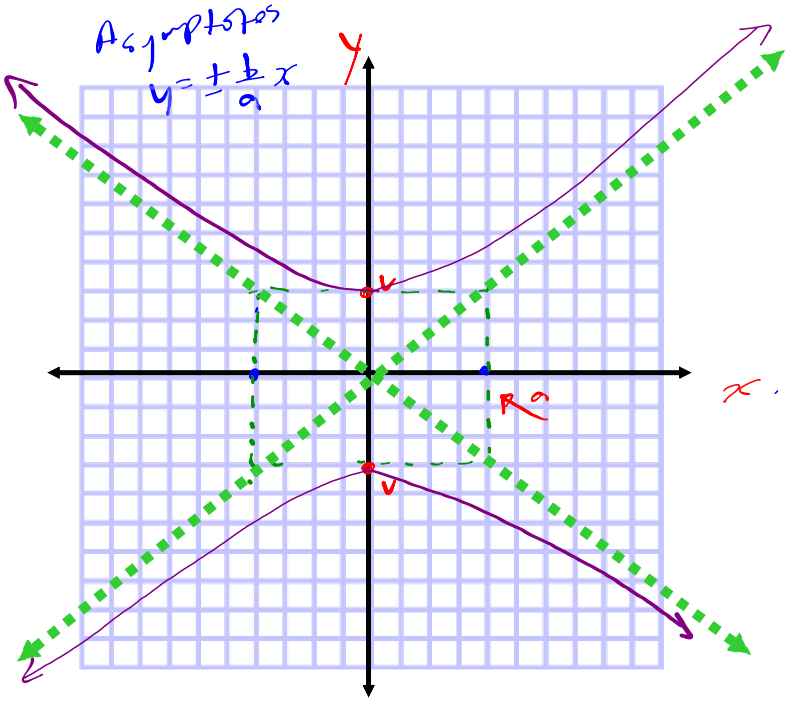
$c^2 = a^2 + b^2$

$c^2 = 16 + 9$

$c = 5$

A: $y = \frac{3}{4}x$

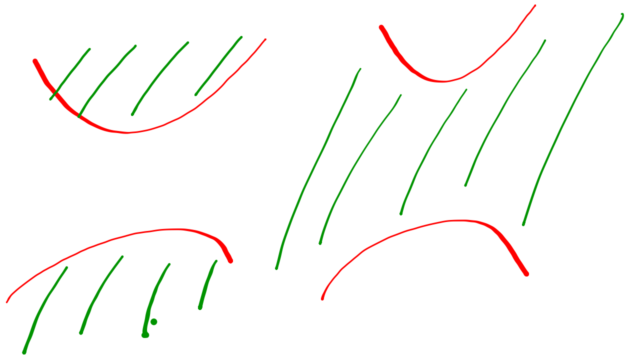
$y = \frac{-3}{4}x$



$\frac{x^2}{9} - \frac{y^2}{25} < 1$

$\frac{y^2}{25} - \frac{x^2}{9} > 1$

For shading, it's either



Step ①: Graph corresponding equality

$\frac{y^2}{25} - \frac{x^2}{9} > 1$

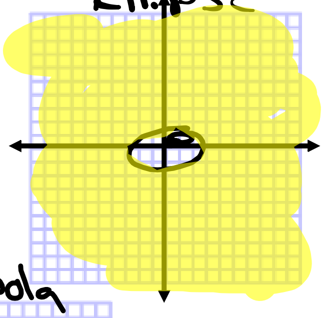
Step ②: For shading, use test (0,0) says Clarence.

Sub test into original inequality to see if it makes a true statement.

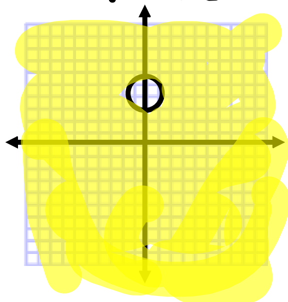
graph $y^2 + \frac{x^2}{4} > 1$	graph $(y-4)^2 + x^2 > 1$	graph $(y-4)^2 > 4x$
graph $y^2 - \frac{x^2}{4} < 1$		

Ellipse

D=R
R=R

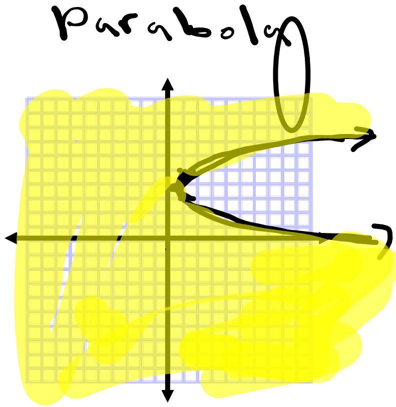


Circle



D=R
R=R
r=1

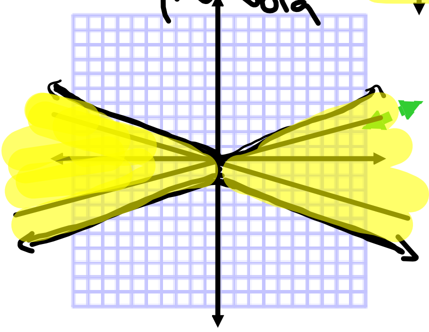
Parabola



D=R
R=R

$$\frac{(y-4)^2}{4} = x$$

Hyperbola



D=R
R=R

Focus=1.73