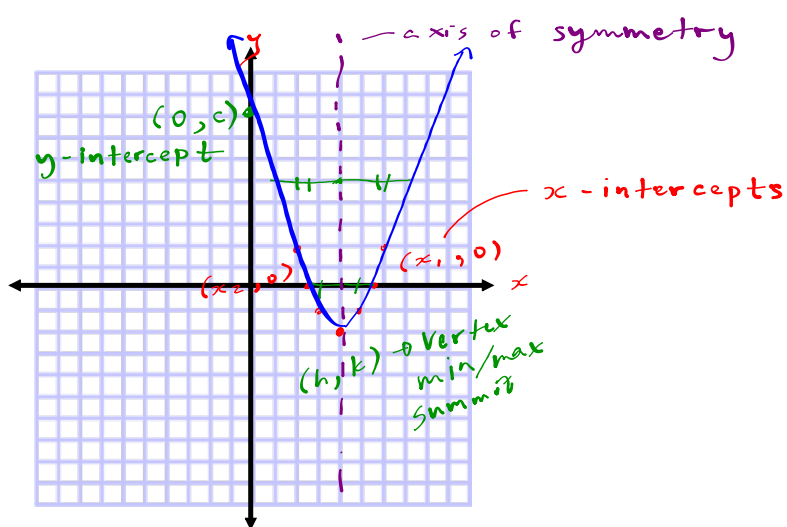
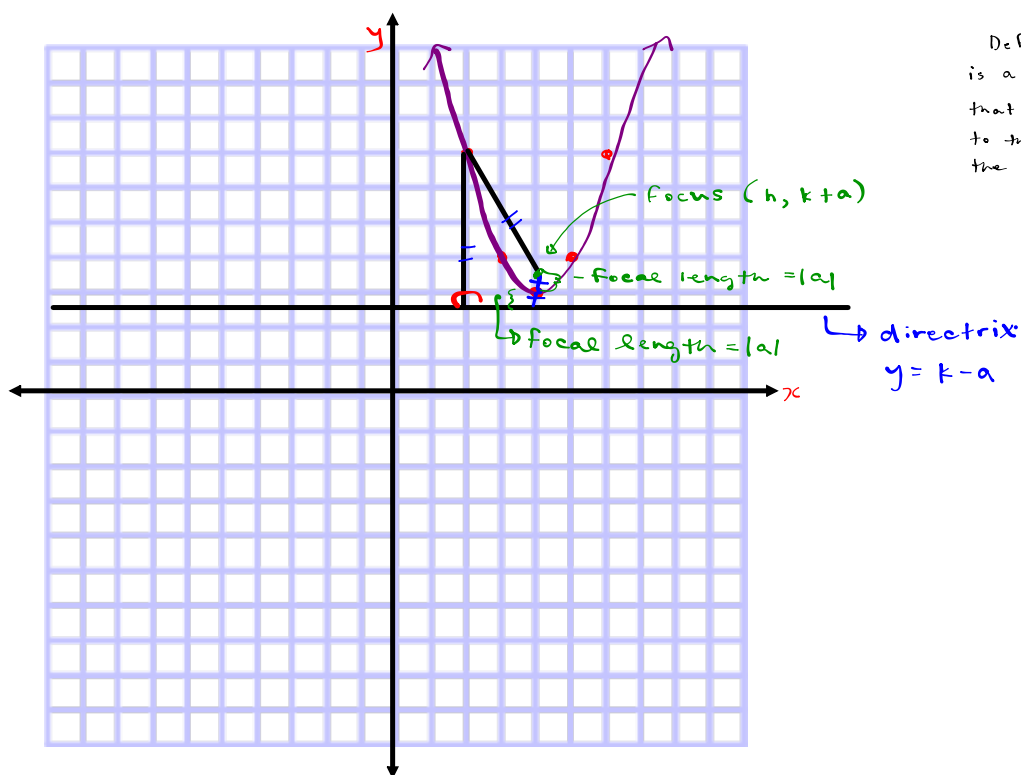


Unit 4 - Standard Form of a Parabola





Definition: A parabola is a set of points that are equidistant to the focus point and the directrix line.

Focus $(h, k+a)$

focal length = $|a|$

focal length = $|a|$

directrix $y = k-a$

Standard form of a parabola facing up or down

$$(x - h)^2 = 4a(y - k)$$

$V(h, k)$ \rightarrow parameter - horizontal translation
 \hookrightarrow parameter - vertical translation

the 'a' parameter corresponds to focal length

a \rightarrow scale factor

if a +, \uparrow if a - \downarrow

ex.

$$(x + 2)^2 = 8(y - 1)$$

$$(x - h)^2 = 4a(y - k)$$

$h = -2$
 $k = 1$
 $\frac{4a}{4} = \frac{8}{4}$
 $a = 2$

ex

$$(x - 1)^2 = 1(y + 2)$$

$$(x - h)^2 = 4a(y - k)$$

$h = 1$
 $k = -2$
 $a = \frac{1}{4}$
 $\frac{1}{4} = \frac{4a}{4}$
 $a = \frac{1}{4}$

ex

$$x^2 = -4y$$

$$(x - h)^2 = 4a(y - k)$$

$h = 0$
 $k = 0$
 $-\frac{1}{4} = \frac{4a}{4}$
 $a = -\frac{1}{4}$

graph

$$(x-3)^2 = -12(y+2)$$

$$(x-h)^2 = 4a(y-k)$$

B
 O
 U
 N
 D
 A
 R
 Y
 S

step i. identify the parameters h/k/a

$h = 3$ $a = -3$ $\frac{4a}{4} = \frac{-12}{4}$
 $k = -2$ $a = -3$

step iii (solving) isolate y in the equation

$$\frac{(x-3)^2}{-12} = \frac{-12(y+2)}{-12}$$

step ii construct table of values and put (h,k) in middle

x	y
h-2	••
h-1	•
h	•
h+1	••
h+2	••

2

x	y
1	-2.33
2	-2.08
3	-2
4	-2.08
5	-2.33

Joel

$$\frac{(x-3)^2}{-12} = y + 2$$

$$\frac{(x-3)^2}{-12} - 2 = y$$

step iv. sub in x-values to calculate y. Plot point draw curve.

$$y = \frac{(1-3)^2}{-12} - 2$$

$$y = \frac{(-2)^2}{-12} - 2$$

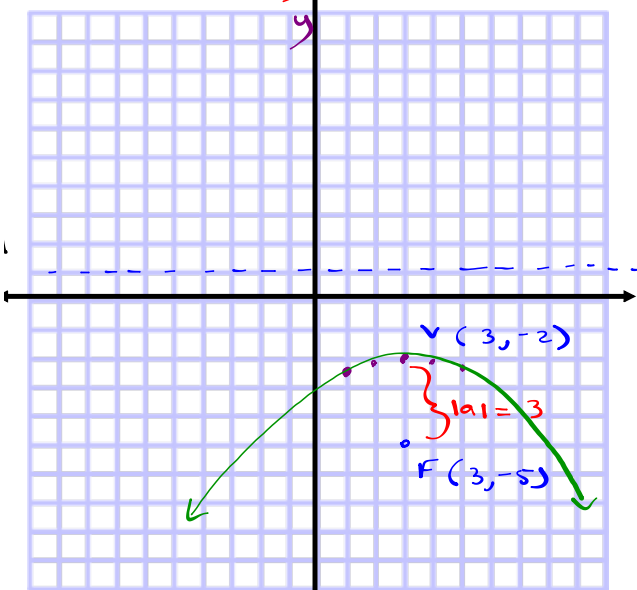
$$y = \frac{4}{-12} - 2$$

$$y = -2.33$$

step v. identify and draw focus and directrix

$F(h, k+a)$
 $F(3, -2 + (-3))$
 $F(3, -5)$

$D: y = k - a$
 $y = -2 - (-3)$
 $y = -2 + 3$
 $y = 1$



Domain $-\infty, \infty$
 Range $-\infty, -2$

graph and state focus / directrix / Domain / Range

$$(x+3)^2 = -(y-2)$$

• Order of Operations

Brackets

Exponents

[Division

 Multiplication

 Addition

 Subtraction

• evaluating

$$y = \frac{(2+2)^2}{4}$$

$$y = \frac{(4)^2}{4}$$

$$y = \frac{16}{4}$$

$$y = 4$$

• solving/isolating

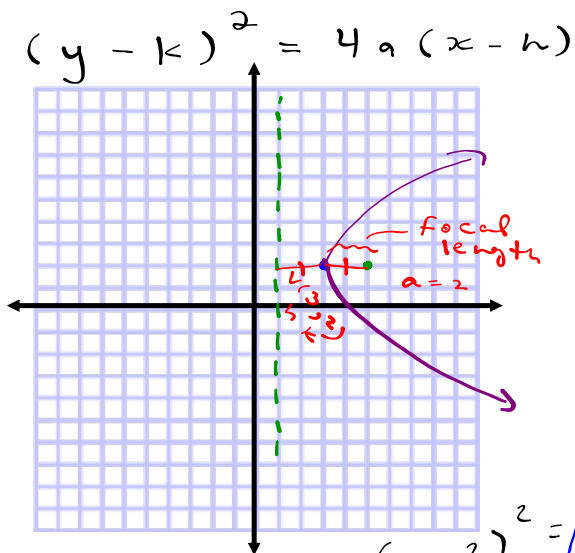
$$4 \cdot 4 = \left(\frac{(x+2)^2}{4} \right) \cdot 4$$

$$\sqrt{16} = \sqrt{(x+2)^2}$$

$$4^{-2} = x + 2$$

$$x = 2$$

Standard Form of a Parabola
facing right or left

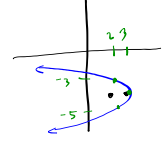


$V(h, k)$
 a - scale factor
 a - focal length
 if $a >$, \curvearrowright
 if $a <$, \curvearrowleft
 Focus : $(h + a, k)$
 Directrix : $x = h - a$

$(y - 2)^2 = 8(x - 3)$
 $(y - k)^2 = 4a(x - h)$

$\frac{4a}{4} = \frac{8}{4}$
 $a = 2$

graph and state characteristics



$$(y + 5)^2 = -8(x - 3)$$

$$(y - k)^2 = 4a(x - h)$$

$h = 3$
 $k = -5$
 $a = -2$

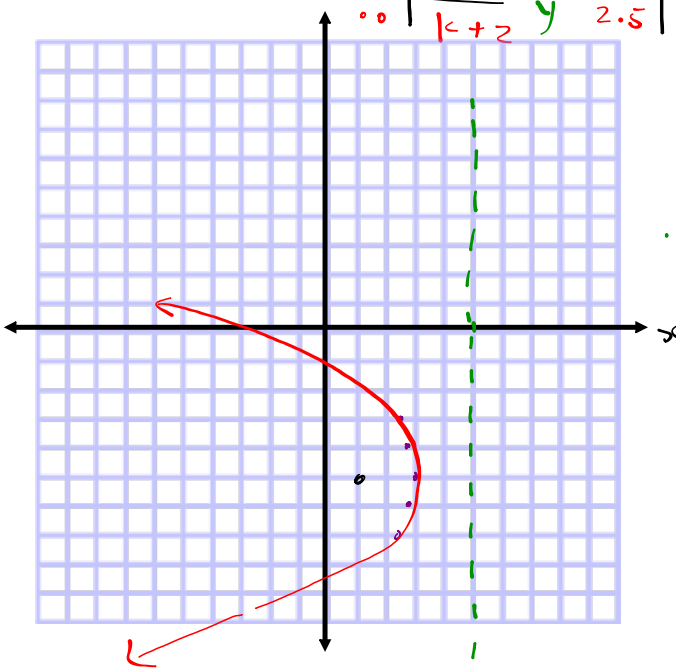
x	y	x	y
$h - 2$	$k - 2$	$h - 2.5$	$k - 3$
$h - 1$	$k - 1$	$h - 2.88$	$k - 4$
h	k	$h - 3$	$k - 5$
$h + 1$	$k + 1$	$h - 2.88$	$k - 6$
$h + 2$	$k + 2$	$h - 2.5$	$k - 7$

isolate x

$$\frac{(y + 5)^2}{-8} = \frac{-8(x - 3)}{-8}$$

$$\frac{(y + 5)^2}{-8} + 3 = x - 3 + 3$$

$$\frac{(y + 5)^2}{-8} + 3 = x$$

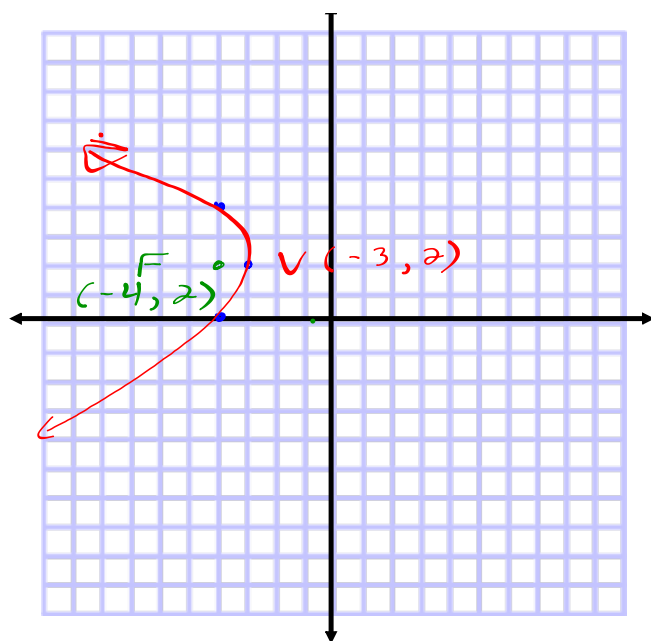


- Focus $F(h + a, k)$
 $F(3 + (-2), -5)$
 $F(1, -5)$

- Directrix $x = h - a$
 $x = 3 - (-2)$
 $x = 5$
- Domain $-\infty, 3\}$
- Range \mathbb{R}

graph and state focus/directrix/domain/range

$$(y - 2)^2 = -4(x + 3)$$



F $|a| = 1$

Find the equation of parabola with focus $(2, 2)$ and the directrix $x = 4$

$$(y - k)^2 = 4a(x - h)$$

$$(y - 2)^2 = -4(x - 3)$$

$$h + a = 2 \quad (1)$$

$$h - a = 4 \quad (2)$$

isolate h in (1)

$$h + a = 2 - a$$

$$h = 2 - a$$

sub (1) into (2)

$$2 - a - a = 4$$

$$-2a = 4 - 2$$

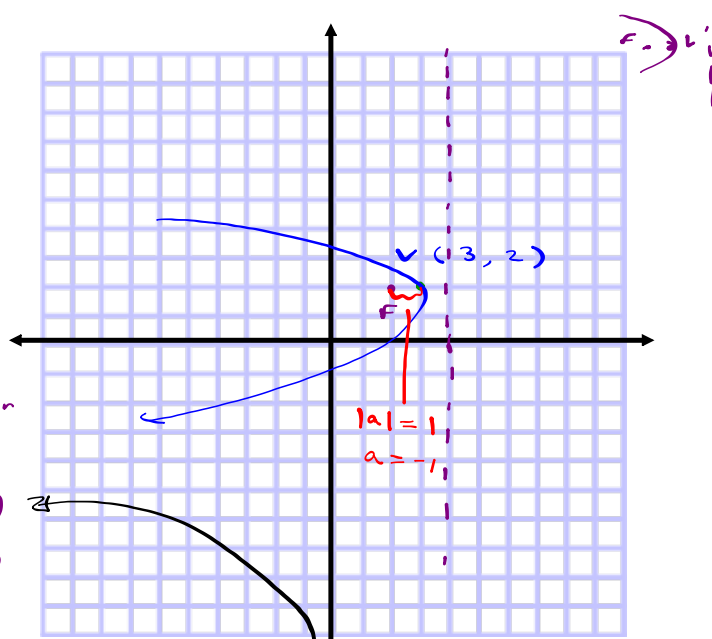
$$-2a = \frac{2}{-2}$$

$$a = -1$$

sub a into (1)

$$h = 2 - (-1)$$

$$h = 2 + 1 \quad h = 3$$

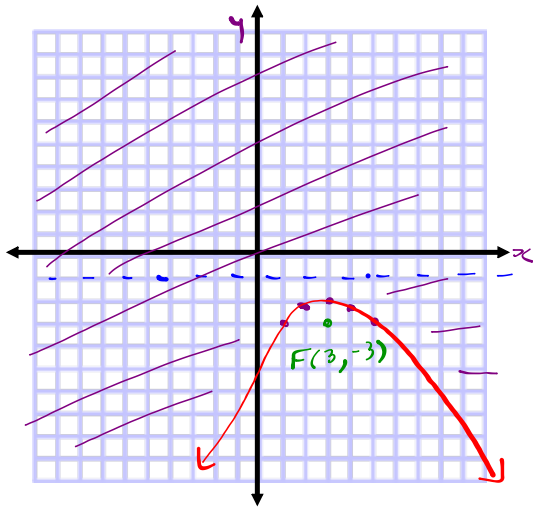


Unit 5: Graphing a Relation
defined by a parabola

① \geq \leq - solid line
② $>$ $<$ - dotted line

ex graph and state all characteristics

$$(x - 3)^2 \geq -4(y + 2)$$



step i. first graph equation

$$(x - h)^2 = 4a(y - k)$$

$a = -1$
 $h = 3$
 $k = -2$

x	y
1	-3
2	-2.25
3	-2
4	-2.25
5	-3

Domain: \mathbb{R}

Range: \mathbb{R}

step ii. For the shading, sub in test point not on parabola (ideally the focus, Stammov):

$$F(3, -3) \quad (3 - 3)^2 \geq -4(-3 + 2)$$

$0 \geq 4$
False shade where point is not.

HWK

P 4.19 - 4.21

P 4.39 #1 - #2

P 5.15 - 5.19

P 5.22 (Domain + Range)
L7 Ex 5.3

graph

$$(y + 2)^2 < -8(x - 2)$$

Find equation of parabola with focus $(-1, -3)$ and directrix $y = 2$