Unit 1: Graphing a Circle

equation → graph

$d = \sqrt{(y_2-y_1)^2+(x_2-x_1)^2}$

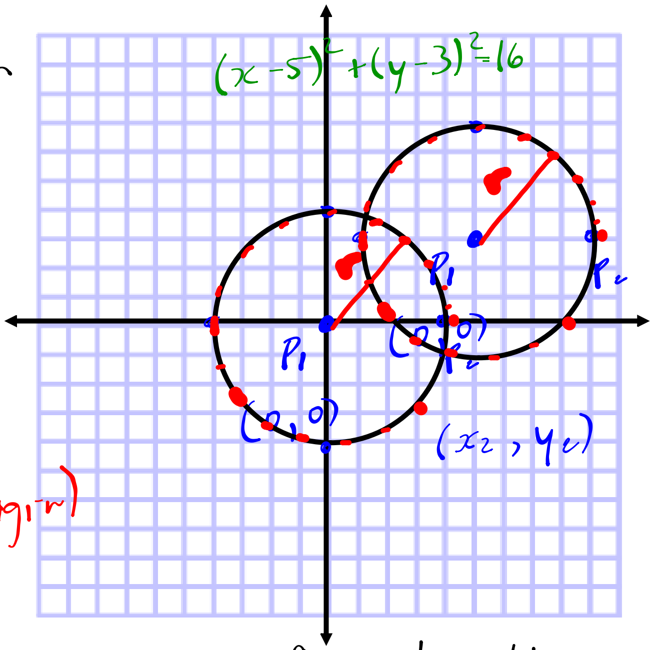
$4 = \sqrt{(y_2-0)^2+(x_2-0)^2}$

$16 = y_2^2 + x_2^2$

$r^2 = y^2 + x^2$ (centred at the origin)

$\sqrt{r^2} = \sqrt{16}$

$r = 4$



Definition of a circle: set of points all equal distance from a centre point.

graph the following conic and state Domain and Range.

$(y-(-2))$

$(x-2)^2 + (y+2)^2 = 4$

$(x-h)^2 + (y-k)^2 = r^2$

Center (h, k)

$(2, -2)$

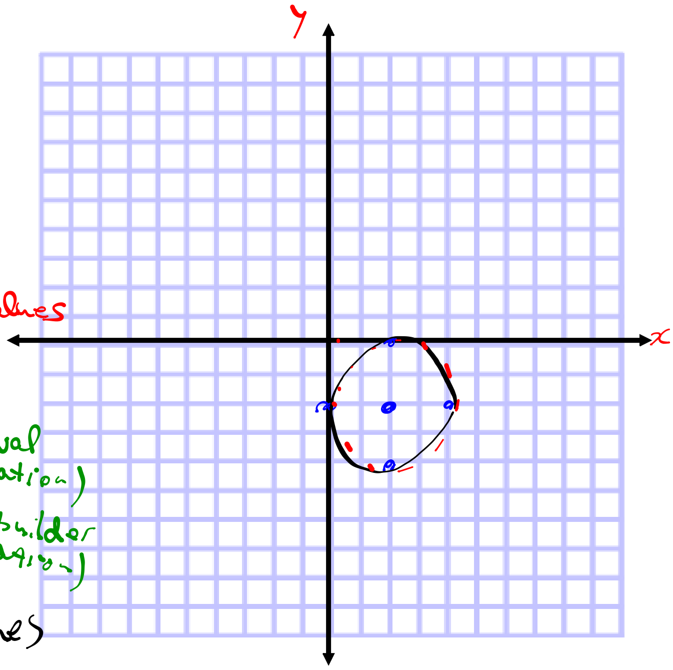
radius $\sqrt{r^2} = \sqrt{4}$

$r = 2$

Domain: the x-values the conic uses.

$[0, 4]$ (interval notation)

$D = \{x \in \mathbb{R} \mid 0 \le x \le 4\}$ (set builder notation)



Range: the y-values the conic uses.

$\{-4, 0\}$

$\{y \in \mathbb{R} \mid -4 \le y \le 0\}$

graph and
state domain and range

$(x+1)^2 + y^2 = 9$

$(x-h)^2 + (y-k)^2 = r^2$

$C(h,k)$ $(-1, 0)$

radius

$r = 3$

$(y+2)^2 + (x-1)^2 = 4$

graph and state D/R

$(y+3)^2 + (x-2)^2 = 9$

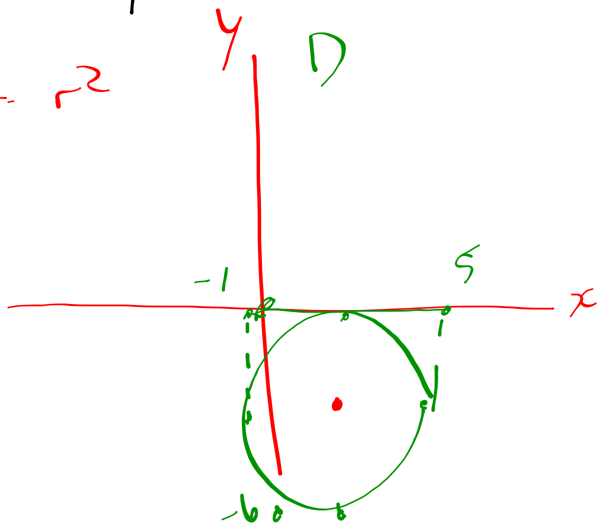
$(x-h)^2 + (y-k)^2 = r^2$

$C(2,-3)$

$r=3$

D $[-1, 5]$

R $[-6, 0]$



graph $(x-4)^2+(y+2)^2=9$

ex $x^2+y^2-8x+4y+11=0$

$x^2+y^2+Dx+Ey+F=0$

$D=-8$

$E=4$

$F=11$

$C\left(-\frac{D}{2},-\frac{E}{2}\right)$ $R=\sqrt{-F+\left(\frac{D}{2}\right)^2+\left(\frac{E}{2}\right)^2}$ $C(h,k)$

radius

$C\left(-\frac{(-8)}{2},-\frac{4}{2}\right)$ $R=\sqrt{-11+\left(\frac{-8}{2}\right)^2+\left(\frac{4}{2}\right)^2}$

$R=\sqrt{-11+16+4}$

$C(4,-2)$ $R=\sqrt{9}$

$R=3$

graph and state D/R

$x^2+y^2-x+4y+4=0$

Converting general form to standard form by "completing the square"

graph

$(+4, -2)$

$r = 3$

$x^2 + y^2 - 8x + 4y + 11 = 0$

$x^2 + y^2 - 8x + 4y = -11$

$x^2 - \underline{8}x + (-4)^2 + y^2 + \underline{4}y + (2)^2 = -11 + (-4)^2 + 2^2$

$\div 2$ $\qquad \div 2$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y+2)^2 = 9$

$(x-4)^2 + (y+2)^2 = -11 + 16 + 4$

$(x-4)^2 + (y+2)^2 = 9$

Complete square (or give it in standard form)

$$x^2 + y^2 - x + 4y + 4 = 0$$

Unit 2: Graphing a Relation (equation or inequality) Defined by a circle

solid line → $\geq$ / $\leq$

dotted line → $>$ / $<$

To graph inequalities

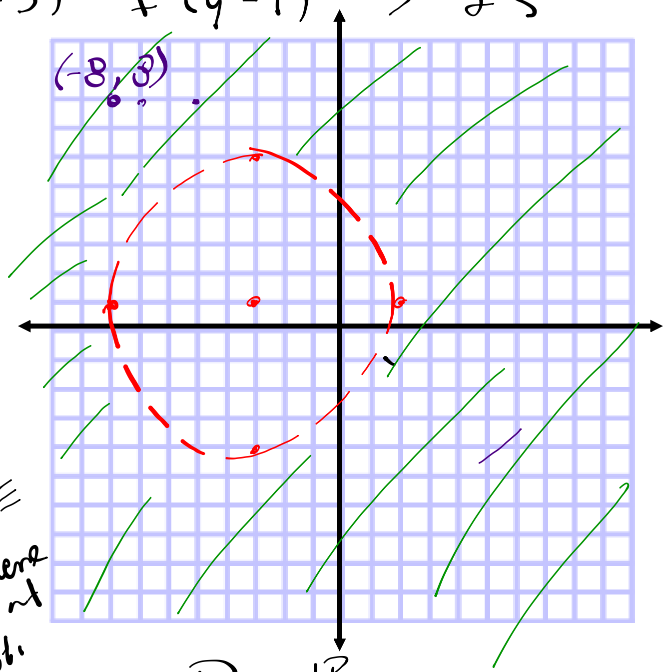
ex. $(x+3)^2 + (y-1)^2 > 25$

step ①: graph corresponding equation

$(x+3)^2 + (y-1)^2 = 25$

$C(-3,1)$

$r\ 5$



Step ②: To know where to shade, sub in a test point (h,k) into original inequality

$(-3, 1)$

$(-3+3)^2 + (1-1)^2 > 25$

$0 > 25$

FALSE

shade where test point is not.

$D = \mathbb{R}$

$D = -\infty, \infty$

$R = \mathbb{R}$

$= -\infty, \infty$

graph and state domain / range

$(x+2)^2 + (y-1)^2 < 9$

$(x-h)^2 + (y-k)^2 = r^2$

$(-2, 1)$

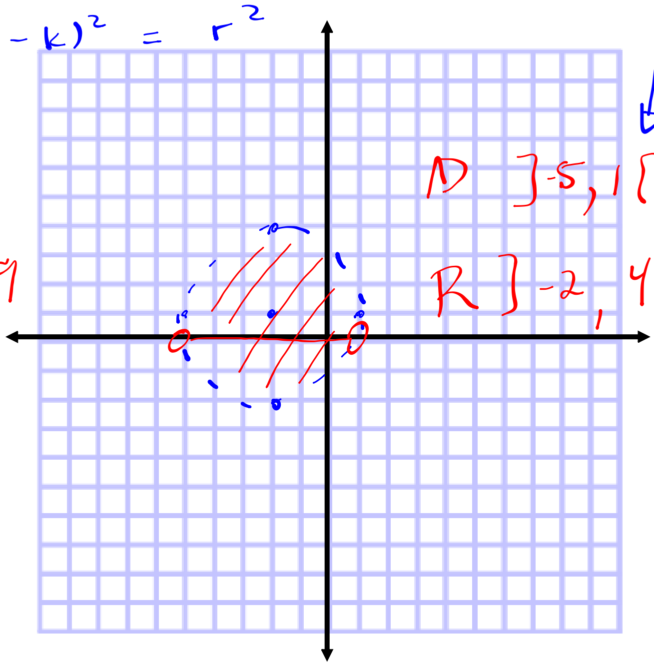
$r = 3$

$(-2+2)^2 + (1-1)^2 < 9$

$0 < 9$

open bracket

I & D



D $]-5, 1[$

R $]-2, 4[$

graph and state Domain and Range

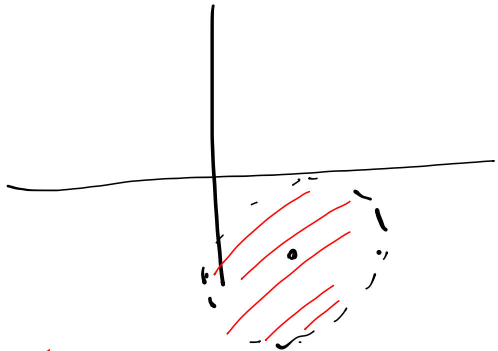
$$x^2+y^2-4x+6y+4<0$$

→ dotted

$$(x-2)^2+(y+3)^2=9$$

C $(2,-3)$

$r=3$



test $(2,-3)$

$$2^2+(-3)^2-4(2)+6(-3)+4<0$$

$$\cancel{4}+9-\cancel{8}-18+\cancel{4}\le 0$$

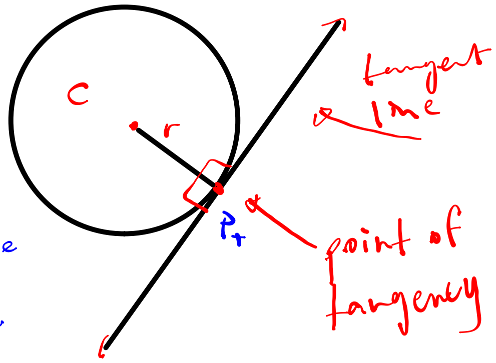
$-9<0$ True!

Unit 3: Finding Equation of a tangent Line to a circle

$y = mx + b$.

perpendicular lines have negative reciprocal slopes.

ex. $\frac{2}{3}$ / $-\frac{3}{2}$ ex. $-\frac{2}{1}$ / $\frac{1}{2}$



tangent line is $\perp$ $\overline{CP_T}$

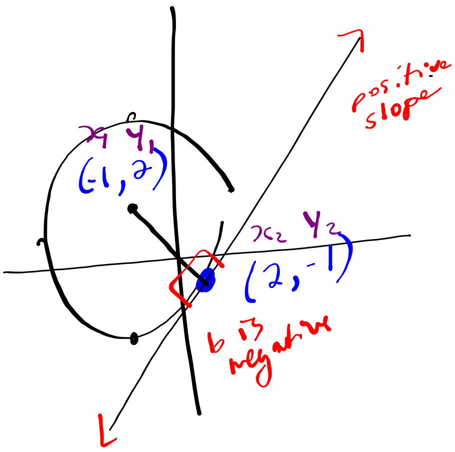
Definition: a tangent is a line that touches the circle at one point.

A line is tangent to a circle at point $(2, -1)$. If the equation of the circle $(x-h)^2 + (y-k)^2 = r^2$ is $(x+1)^2 + (y-2)^2 = 18$, determine the equation of the tangent line. $y = mx + b$

Circle

$(-1, 2)$

$r = \sqrt{18} = 4.24$



Step ① First find m_{radius}. Take negative reciporcal. ($m_{Tangent}$)

$$m_r = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_r = \frac{-1 - 2}{2 - (-1)}$$

$$m_r = \frac{-3}{3}$$

$$m_r = \frac{-1}{1} \rightarrow m_T = 1$$

radius ⊥ tagent

Step ② Find the 'b', by subbing in temporarily the point of tanreny

$y = 1x + b$

sub $(2, -1)$

$-1 = 1(2) + b$

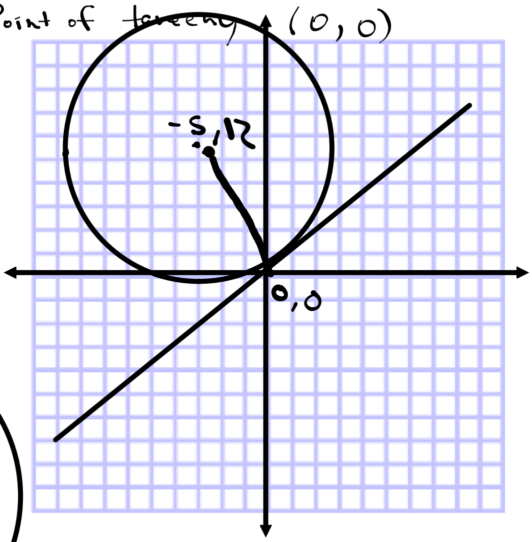
$b = -3$

$y = mx + b$

$$\boxed{y = x - 3}$$

Find the equation of the tangent to circle $(x+5)^2+(y-12)^2=169$.

Point of tangency (0, 0)



$m=\frac{12-0}{-5-0}=\frac{12}{-5}$ $m_t=\frac{5}{12}$

$y=mx+b$

$\frac{5}{12}=\frac{y-0}{x-0}$

$12y=5x$

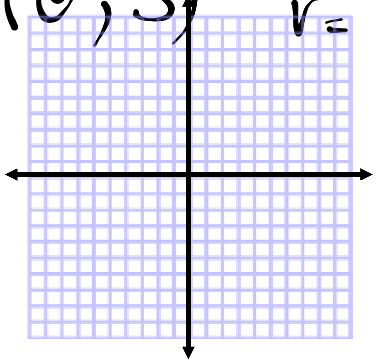
$y=\frac{5}{12}x$

Find the equation of the tangent line that passes through the circle $x^2+y^2-6y-16=0$ at point $(3,7)$

$x^2+(y-3)=16+(3)^2$

$x^2+(y-3)=25$

$(0,3)$ $r=5$



$(3,7)$

$(0,3)$

$m=\frac{7-3}{3-0}=\frac{4}{3}$

$m_2=-\frac{3}{4}$

$-\frac{3}{4}=\frac{y-7}{x-3}$

$4y-28=-3x+9$

$\boxed{y=\frac{-3}{4}x+\frac{37}{4}}$