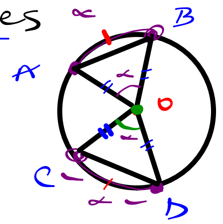
Unit 9: Proofs Containing Circles and Triangles

SSS
SAS
ASA

Prove: Th 70. Congruent arcs
arcs subtended by congruent chords.



Hypothesis: (what you know is true)

$\overset{\frown}{AB} \cong \overset{\frown}{CD} = \alpha$

O. origin

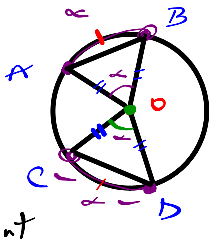
Conclusion to be proved: (what you're proving is true)

$\overline{AB} \cong \overline{CD}$

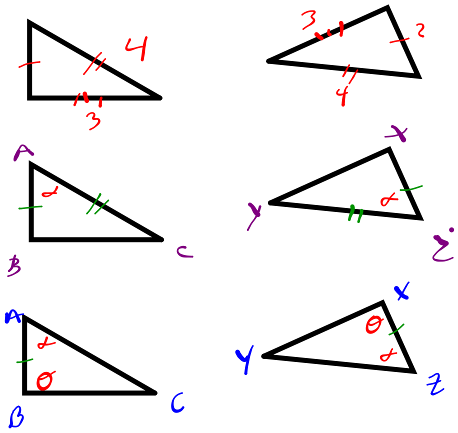
strategy
- construct Δ's
- congruent Δ's
- right Δ's
- Δ connected to origin
- similar Δ's
- think about/label the different angles in a circle
  - central angle
  - inscribed
  - exterior
  - interior

arcs

Statements	Justification
s $\overline{AO} \cong \overline{OC}$	radii in circle
s $\overline{OB} \cong \overline{OD}$	" "
$\angle AOB = m\overset{\frown}{AB}$	Th 76 (central angle theorem)
$\angle COD = m\overset{\frown}{CD}$	" "
since $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ then $\angle COD \cong \angle AOB$	hypo
a $\Delta AOB \cong \Delta COD$	SAS Th 16
1 $\overline{AB} \cong \overline{CD}$	Cuz corresponding sides are congruent in congruent Δ's Th 39 a)



How to prove 2 Δ's are congruent

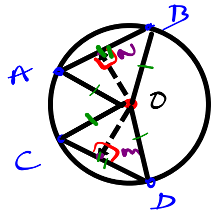


Th 15
if
SSS ≅ SSS, then Δ ≅ Δ

Th 16
if
SAS ≅ SAS, then Δ ≅ Δ

Th 17
if
ASA ≅ ASA, then Δ ≅ Δ

Prove Th 22



SSS
SAS
ASA

strategy
· construct Δ's
· congruent Δ's
· right Δ's
· Δ connected to origin
· similar Δ's

· think about/label the different angles in a circle
angles: · central angle
· inscribed
· exterior
· interior

· strategic lines

Hypothesis:

$\overline{AB} \cong \overline{CD}$

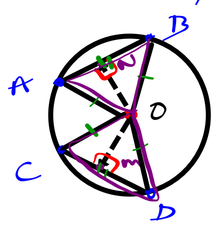
O - origin

$\angle ONA \cong \angle OMC = 90°$

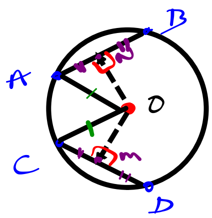
Conclusion: $\overline{ON} \cong \overline{OM}$

Statements	Justification
$\overline{AO} \cong \overline{OC}$	radii
$\overline{OB} \cong \overline{OD}$	''
$\overline{AB} \cong \overline{CD}$	hypo
$\triangle AOB \cong \triangle OCD$	Th 15 SSS
$\overline{OM} \cong \overline{ON}$	39 a) height

SSS



Prove Th 72



Hypothesis :

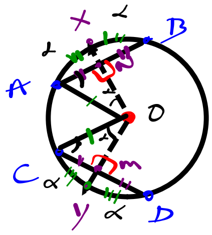
$\overline{AB} \cong \overline{CD}$

O - origin

$\angle ONA \cong \angle OMC = 90°$

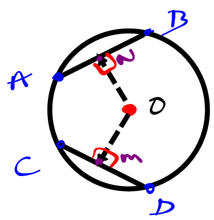
Conclusion: $\overline{ON} \cong \overline{OM}$

Statements	Justification
S $\overline{OC} \cong \overline{AO}$	radii
$\overline{AB} \cong \overline{CD}$	hypo
S $\overline{AN} \cong \overline{NB} \cong \overline{CM} \cong \overline{MD}$	Th 71
$\angle ANO \cong \angle CMO = 90°$	hypo
$\overset{\frown}{AX} \cong \overset{\frown}{XB} \cong \overset{\frown}{CY} \cong \overset{\frown}{YD} = \alpha$	Th 71
$\angle AON \cong \angle COM = \alpha$	Th 76
$\angle OAN = 180° - 90° - \alpha$	180° in a $\triangle$
$\angle MCO = 180° - 90° - \alpha$	
A $\therefore \angle OAN \cong \angle MCO$	SAS
$\triangle COM \cong \triangle AON$	39 a)
$\overline{OM} \cong \overline{ON}$	



SAS

Prove Th 22



Hypothesis:

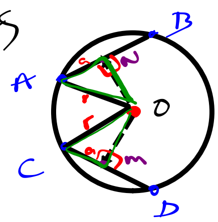
$\overline{AB} \cong \overline{CD}$

O – origin

$\angle ONA \cong \angle OMC = 90^\circ$

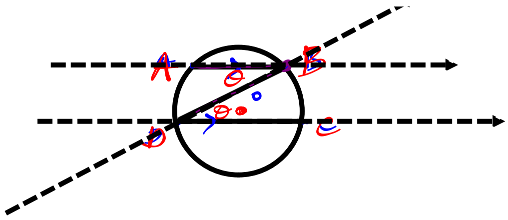
Conclusion: $\overline{ON} \cong \overline{OM}$

Statements	Justification
$\overline{ON} = \sqrt{r^2 - a^2}$	Pythagorous
$\overline{OM} = \sqrt{r^2 - a^2}$	



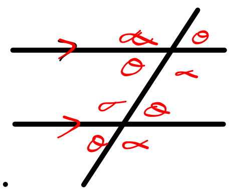
$\therefore \overline{ON} = \overline{OM}$

Th 79:
Prove that ∥ lines intercept congruent arcs.



Hypothesis:

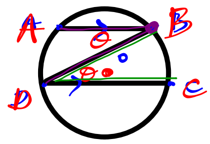
$\overline{AB} \parallel \overline{DC}$
• O origin



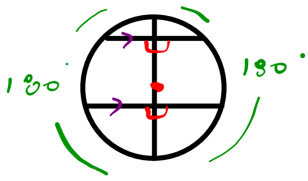
Conclusion: $\overset{\frown}{AD} \cong \overset{\frown}{BC}$

• think about/label the different angles in a circle
arcs { • central angle
• inscribed
• exterior
• interior
• strategic lines

S	J
$\angle ABD = \angle BDC = \theta$	Th 3 a)
$\angle ABD = \frac{1}{2}\overset{\frown}{AD}$ $2\theta = \left(\frac{1}{2}\overset{\frown}{AD}\right)2$ $\overset{\frown}{AD} = 2\theta$	Th 77
$\angle BDC = \frac{1}{2}\overset{\frown}{BC}$ $2\theta = \frac{1}{2}\overset{\frown}{BC}\,2$ $\overset{\frown}{BC} = 2\theta$ $\therefore \overset{\frown}{AD} = \overset{\frown}{BC}$ QED	



Th 74:
Prove that ∥ lines intercept congruent arcs.



Hypothesis:

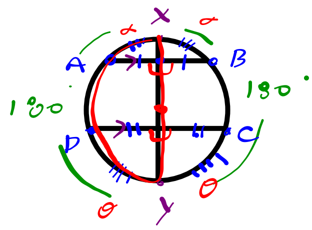
$\overline{AB} \parallel \overline{DC}$

• O origin

Conclusion: $\widehat{AD} \cong \widehat{BC}$

• think about/label the different angles in a circle
arcs: • central angle • inscribed • exterior • interior

Statements	Justification.
diameter	
$\widehat{XAY} = \widehat{XBY} = 180°$	diameter
$\widehat{AX} \cong \widehat{XB} = \alpha$	Th 71
$\widehat{DY} \cong \widehat{YC} = \theta$	Th 71
$\widehat{AD} = 180° - \alpha - \theta$	
$\widehat{BC} = 180 - \alpha - \theta$	
$\widehat{AD} = \widehat{BC}$	

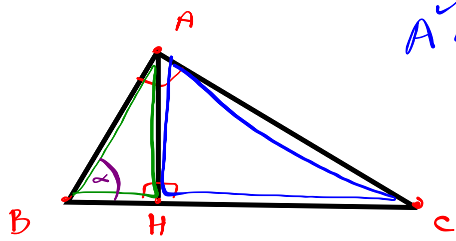


Th 90

Hypothesis:

$\angle BHA = 90°$

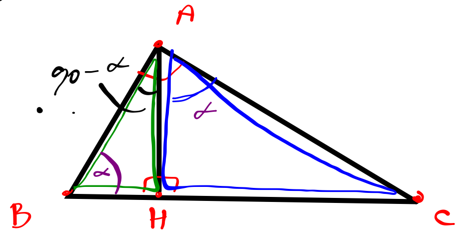
$\angle BAC = 90°$



$A\hat{\ } A \cong \hat{A}\hat{A}$

Conclusion to be proved: $AH^2 = BH \times HC$.

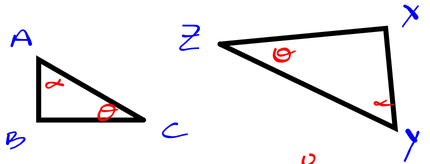
Statements	Justification
a $\angle BHA \cong \angle AHC = 90°$	hypo
$\angle BAH = 180° - 90° - \alpha$ $\angle BAH = 90 - \alpha$	180° in Δ
$\angle HAC = 90° - (90° - \alpha)$ $\angle HAC = 90 - 90 + \alpha$ $\angle HAC = \alpha$	complementary angles (adds up to 90°)
a $\angle ABH = \angle HAC = \alpha$	
$\Delta ABH \sim \Delta AHC$	AA th 18
$\frac{\overline{BH}}{\overline{AH}} = \frac{\overline{AH}}{\overline{CH}}$	Th 50 a)
$\overline{BH} \times \overline{CH} = \overline{AH} \times \overline{AH}$	
$\overline{BH} \times \overline{HC} = \overline{AH}^2$	



$AH^2 = BH \times HC$

How to prove 2 Δ's are similar

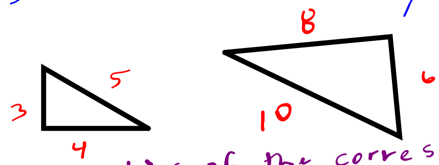
Th 18



if $\boxed{AA \cong AA}$, then Δ ~ Δ

$\frac{6}{3} = 2$ $\frac{8}{4} = 2$ $\frac{10}{5} = 2$

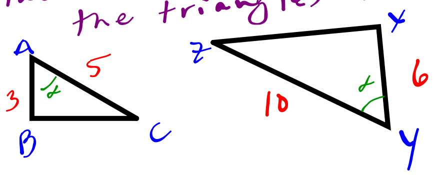
Th 19



if $\frac{S_s}{S_s} = \frac{S_m}{S_m} = \frac{S_B}{S_B}$, then Δ ~ Δ

↳ if the ratios of the corresponds (all of them) are equal, then the triangles are similar.

Th 20



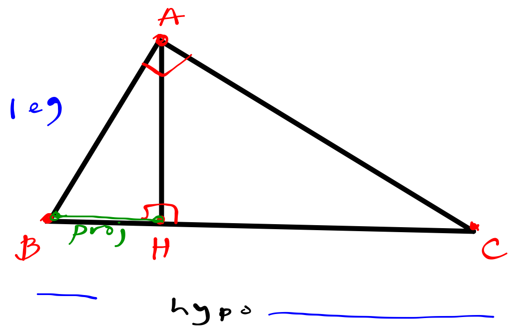
if $\frac{S_s}{S_s} = \frac{S_m}{S_m}$ $A \cong A$, then Δ ~ Δ

∠ contained angle

$\frac{XZ}{BC} = \frac{S_s}{S_s}$ Th 50

Prove Th 8 9

Hypo :



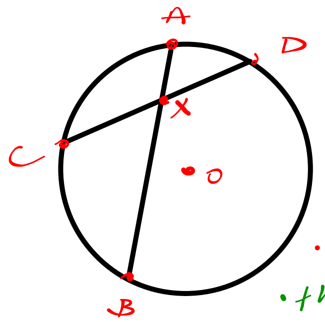
Conclusion to be proved :

S	J

Prove Th 81

hypo

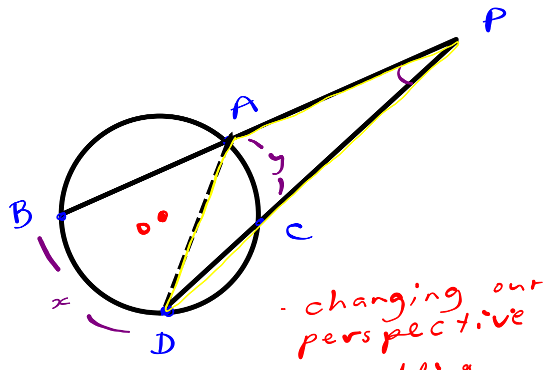
Conclusion to be proved:



· think about/label the different angles in a circle

angles
· central angle
· inscribed
· exterior
· interior

Prove Th 79



Hypo:

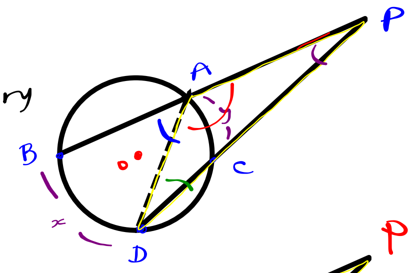
- O − origin
- $\overset{\frown}{BD} = x$
- $\overset{\frown}{AC} = y$

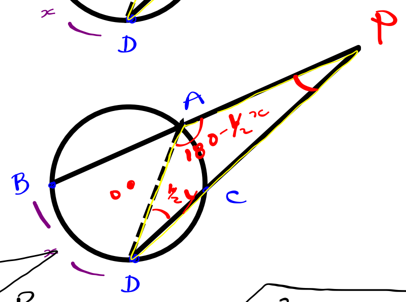
Conclusion: $\angle P = \frac{1}{2}(x - y)$

· changing our perspective (by adding smtg)

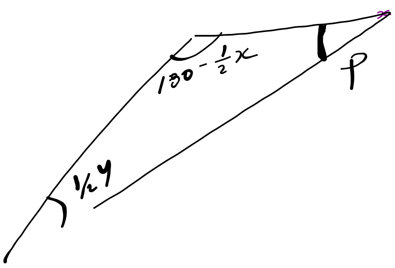
Statements	Justification
$\angle ADC = \frac{1}{2}y$	Th 77
(2) $\angle BAD = \frac{1}{2}x$	Th 77
(3) $\angle DAP = 180 - \angle BAD$	supplementary angles (180)
$\angle DAP = 180 - \frac{1}{2}x$	substitution (2) + (3)
$\angle P = 180 - (180 - \frac{1}{2}x) - \frac{1}{2}y$	180° in a △
$\angle P = \cancel{180 - 180} + \frac{1}{2}x - \frac{1}{2}y$	
$\angle P = \frac{1}{2}(x - y)$	

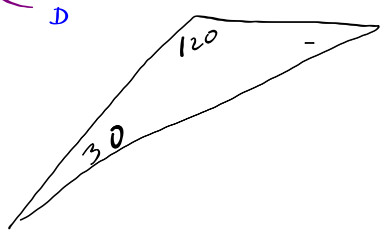
for example
· ∠P is an exterior angle
· what else is it?
∠P is also an angle in a △.





∎





#3 on p 9.8

#3 p 9.15

p 9.17 − 9.23

#2 on p 9.7