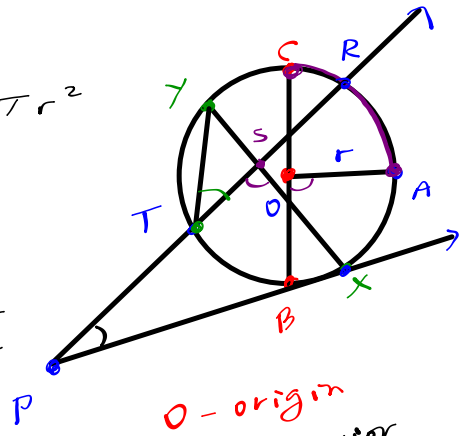
Unit 8: Circles and their Theorems

Recall: elements in circle.



$\overline{OA}$ = radius

$\overline{BC}$ = diameter = $2r$

$\overset{\frown}{AC}$ = arc

$\overline{xy}$ = chord

Circles

$\overline{PR}$ = secant (through circle at two points)

$\overline{PX}$ = tangent (touches circle at one point)

Area of circle $= \pi r^2$

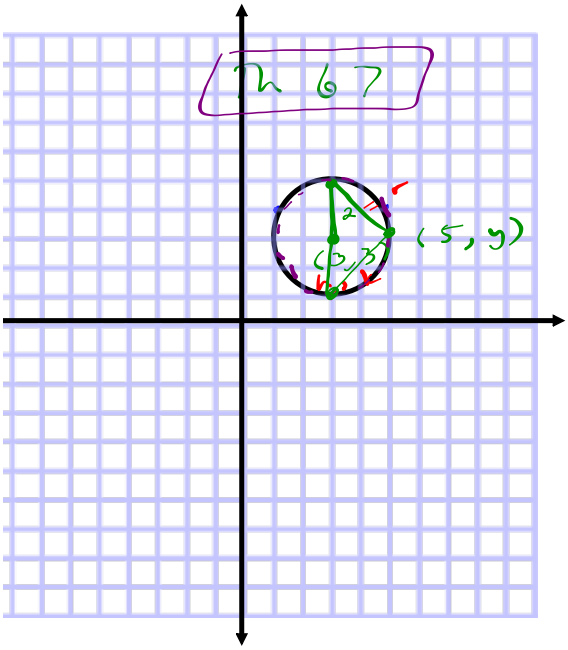
$\angle TSX$ - interior angle

$\angle AOB$ - central angle

O - origin

$\angle RPX$ - exterior angle

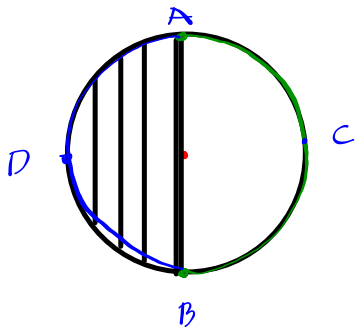
$\angle YTR$ - inscribed angle



$(x-h)^2 + (y-k)^2 = r^2$

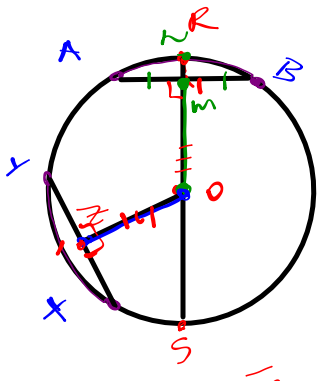
$(x-3)^2 + (y-3)^2 = 4$

Th 68 / Th 69



$m\overset{\frown}{ACB} \cong m\overset{\frown}{ADB} = 180°$

Th 70



if $\overset{\frown}{AB} \cong \overset{\frown}{XY}$

then $\overline{AB} \cong \overline{XY}$

Th 71

if $\overline{RS}$ ⊥ $\overline{AB}$

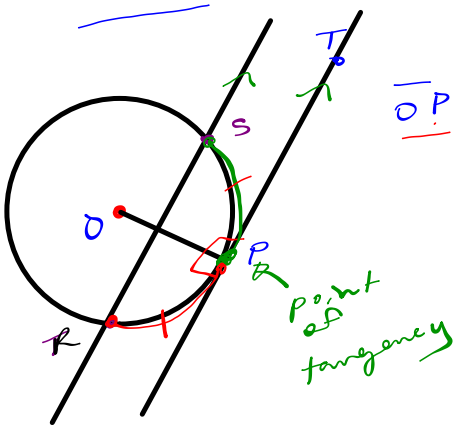
then $\overline{AM} \cong \overline{MB}$

& $\overset{\frown}{AN} \cong \overset{\frown}{NB}$

Th 72 if $\overline{YX} \cong \overline{AB}$

then $\overline{OZ} \cong \overline{OM}$

Th 73



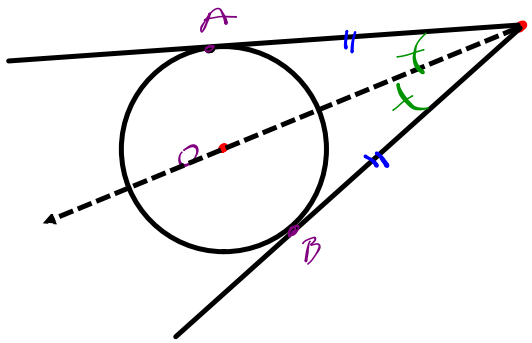
$\overline{OP} \perp \overline{TP}$

Th 74

if $\overline{SR} \parallel \overline{TP}$

then $\hat{SP} \cong \hat{RP}$

Th 7 5

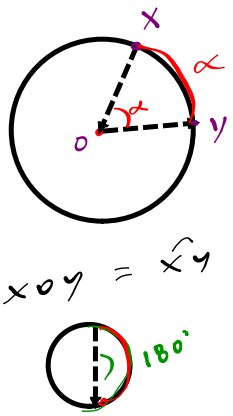


: if two tangent segments share same point P

then $\overline{PA} \cong \overline{PB}$

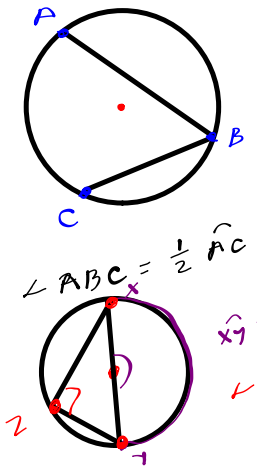
$\angle APO \cong \angle OPB$

Th 76



$\angle XOY = \overset{\frown}{XY}$

Th 77

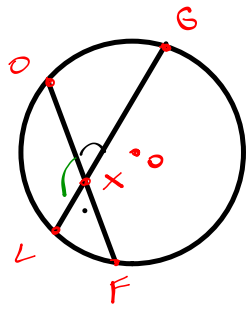


$\angle ABC = \frac{1}{2}\overset{\frown}{AC}$

$\overset{\frown}{XY} = 180$

$\angle XZY = \frac{1}{2}(180) = 90°$

Th 78

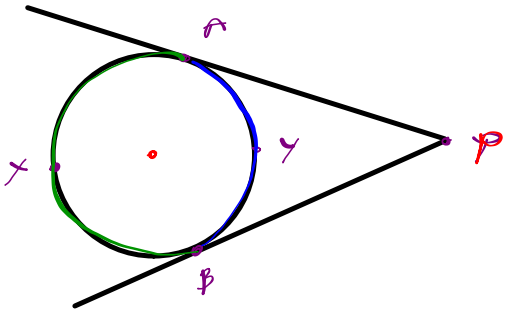


$\angle OXG = \frac{1}{2}(\overset{\frown}{OG} + \overset{\frown}{LF})$

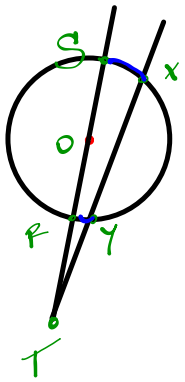
$\angle LXF = \frac{1}{2}(\overset{\frown}{LF} + \overset{\frown}{OG})$

$\angle OXL = \frac{1}{2}(\overset{\frown}{GF} + \overset{\frown}{OL})$

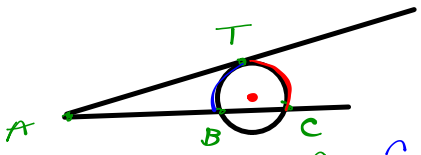
Th 79



$\angle P = \frac{1}{2}(\overset{\frown}{AXB} - \overset{\frown}{AYB})$

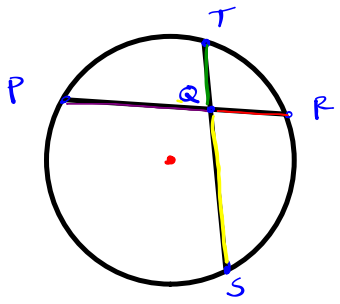


$\angle STX = \frac{1}{2}(\hat{SX} - \hat{PY})$



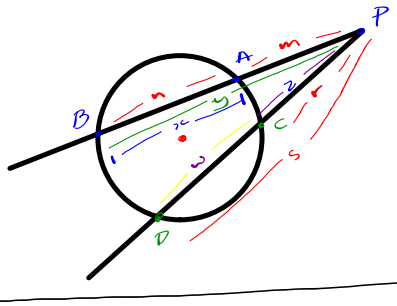
$\angle A = \frac{1}{2}(\hat{TC} - \hat{TB})$

Th B1 (new)



$\overline{PQ} \times \overline{QR} = \overline{TQ} \times \overline{QS}$

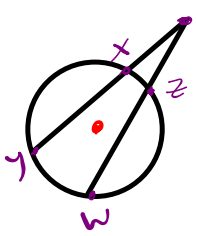
Th 32 (new)



$m\overline{PA} \times m\overline{PB} = m\overline{PC} \times m\overline{PD}$

$m \times (m+n) = r \times s$

$(y-x) \times y = z \times (z+w)$

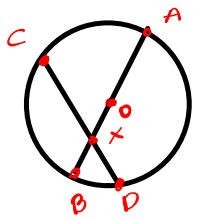


$\overline{PY} = 12.75$

$\overline{PW} = 16.5$

$\overline{PX} = 5.5$

Find $\overline{PZ}$

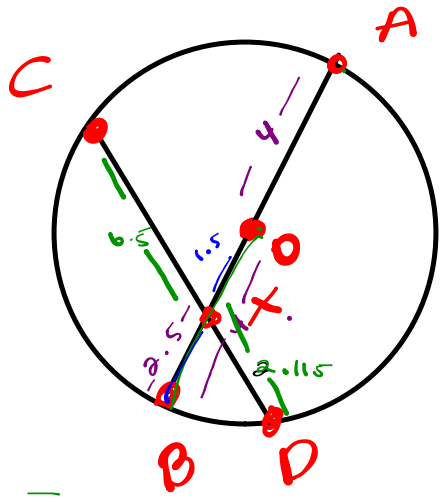


$\overline{OA} = 4$ units

$\overline{BX} = 2.5$ units

$\overline{CX} = 6.5$ units

Find $\overline{CD}$



$\overline{CD} = \overline{CX} + \overline{XD}$

$\overline{CD} = 6.5 + 2.115$

$\overline{CD} =$

$\overline{OA} = 4$ units

$\overline{BX} = 2.5$ units

$\overline{CX} = 6.5$ units

find $\overline{CD}$

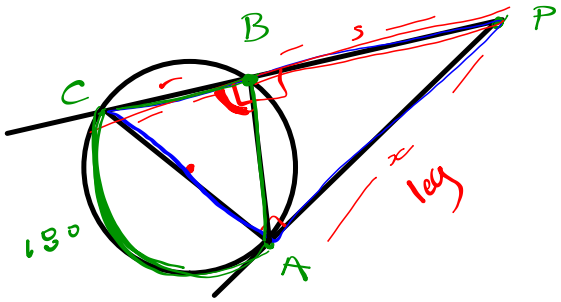
Th B1

$\overline{AX} \times \overline{XB} = \overline{CX} \times \overline{XD}$

$\frac{5.5 \times 2.5}{6.5} = \frac{6.5 \times \overline{XD}}{6.5}$

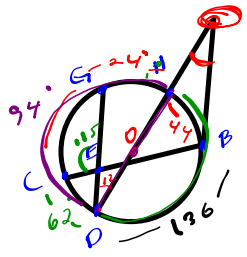
$\overline{XD} = 2.115$

Pr 83



$\therefore \overline{PA}^2 = \overline{PB} \times \overline{PC}$

$x^2 = s \times (s+r)$



$\angle GDH = 12°$

$\overset{\frown}{CD} = 62°$

$\angle CEO = 115°$

find $\angle DAB$ and $\overset{\frown}{CG}$

$\angle GDH = \frac{1}{2}\overset{\frown}{GH}$

$\frac{12}{1/2} = \frac{\frac{1}{2}\overset{\frown}{GH}}{1/2}$

$24 = \overset{\frown}{GH}$

$\overset{\frown}{CG} = 180 - 24° - 62°$

$\overset{\frown}{CG} = 94°$

$\angle CEG = \frac{1}{2}(\overset{\frown}{CG} + \overset{\frown}{DB})$

$\frac{115}{1/2} = \frac{1}{2}(94 + \overset{\frown}{DB})$

$230 = 94 + \overset{\frown}{DB}$

$-94 \quad -94$

$\overset{\frown}{DB} = 136°$

- LABEL
- identify elements you know the value of
  - chord + segment

pick a theorem w those elements are write it out before subbing in values.

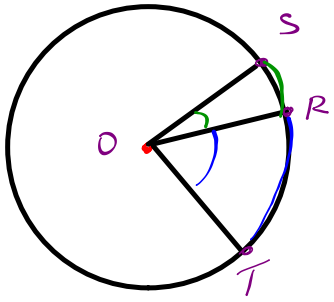
- solve by doing opposite operation
- conceptualize.

$\angle DAB = \frac{1}{2}(\overset{\frown}{DB} - \overset{\frown}{HB})$

$= \frac{1}{2}(136° - 44°)$

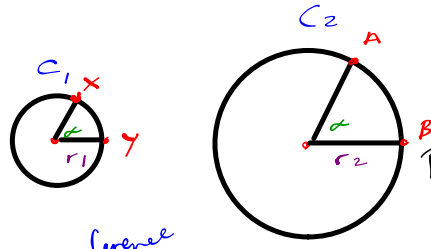
$\angle DAB = 46°$

Th 8 4



$$\frac{\angle SOR}{\angle ROT} = \frac{\overset{\frown}{SR}}{\overset{\frown}{RT}}$$

Th 86



Circumference

Th 86 $\frac{C_1}{C_2} = \frac{r_1}{r_2}$

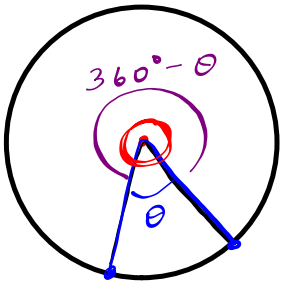
the ratio of the circum... is equal to the ratio of the radii

Th 88 $\frac{\hat{XY}}{\hat{AB}} = \frac{r_1}{r_2}$

Th 87 $\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

Sectors in a Circle and Their Area (portion)

Definition: sector: a part of a circle formed by two radii.



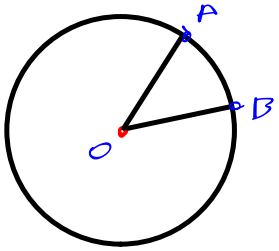
θ is always a central angle by definition.

Th 85 $\frac{\text{Area}_{\text{sector B}}}{\text{Area}_{\text{sector P}}} = \frac{\theta}{360^\circ - \theta}$

Nota Bene! To find the area of a given sector, make the sector in Th 85 the total sector/circle

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\theta}{360^\circ}$$

$$\text{Area of sector} = \frac{\pi r^2 \cdot \theta}{360^\circ}$$

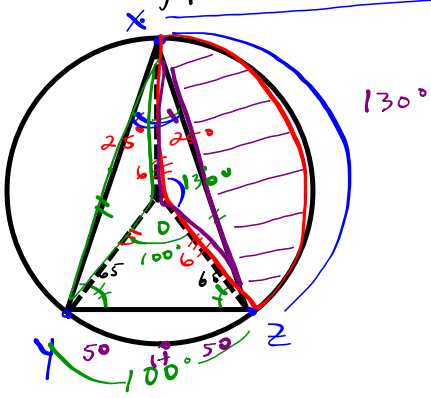


if radius = 4 units

$\angle AOB = 35^\circ$

find area of minor sector AOB.

Typical Exam Calibre Question

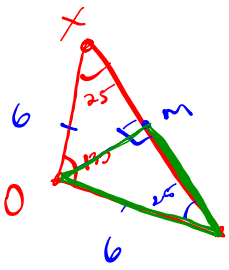


if ΔXYZ is isosceles and $\angle YXZ = 50°$ and inscribed in a circle if radius 6 units, find the area of the shaded region.

$A_{SR} = A_{sector} - A_{\Delta XOZ}$

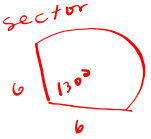
base height

$A_\Delta = \frac{b \times h}{2}$



$\angle Y = \frac{180° - 50°}{2}$

sector



$A_{sector} = \pi r^2 \cdot \frac{\theta}{360°}$

$A_{sector} = \pi 6^2 \cdot \frac{130}{360}$

$A_{sector} = 40.84 \text{ units}^2$

(use total area for 2nd sector)

to find height

$\sin\theta = \frac{opp}{hypo}$

$\sin 25 = \frac{\overline{Om}}{6}$

$\overline{Om} = 6 \sin 25°$

$= 2.536$

to find half of base

$\cos\theta = \frac{adj}{hypo}$

$\cos 25 = \frac{\overline{Zm}}{6}$

$\overline{Zm} = 6 \cos 25$

$\overline{Zm} = 5.438$

$A_\Delta = \frac{\overline{Om} \times \not{2}\,\overline{Zm}}{\not{2}}$

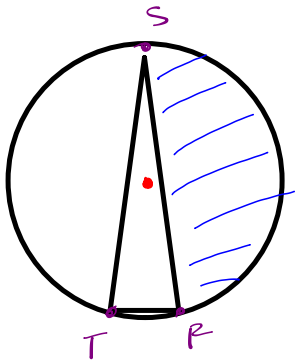
$A_\Delta = (6 \sin 25)(6 \cos 25)$

$A_\Delta = 13.79 \text{ units}^2$

$A_{SR} = A_{sector} - A_\Delta$

$= 40.84 - 13.79$

$A_{SR} = 27.05 \text{ units}^2$



$\angle S = 20°$

radius = 5 units.

$\triangle STR$ – isosceles

Find area of shaded region.

ANS 30.63 units^2

#5 pg 8.13

p 8.33 – 8.42*

HmWk
Ex 3 pg 8.7
#1 p 8.9
8.10 – 8.11
#2 – 3