Unit 1. perations on Real Functions $(+, -, \times, \div)$

Adding 2 Functions

$\boxed{(f+g)(x) = f(x) + g(x)}$

old if $f(x) = \sqrt{x-2} + 4$

old $g(x) = -x - 4$

find $f+g$

$f+g(x) = \sqrt{x-2} + 4 + (-x-4)$

ANS $\boxed{f+g(x) = \sqrt{x-2} - x}$ A

(new)

step ①: sub in the value of the functions and perform operation.

+ ~~+~~ adding like terms.

step ②: Find the domain by considering any restrictions on the new function and old functions.

Domain: $x - 2 + 2 \geq 0 + 2$

Domain: $x \geq 2$

$D: \{x \in \mathbb{R} \mid x \geq 2\}$ set b notation

$[2, \infty$ interval notation.

To find the domain, consider any restrictions:

Ex I: square root function

$y = a\sqrt{b(x-h)} + k$

Domain $b(x-h) \geq 0$

isolate x!

(inequality switches when dividing by negative #)

ex find the domain

$y = 2\sqrt{-(x+3)} + 2$

$\frac{-(x+3)}{-1} \geq \frac{0}{-1}$

$(x+3) - 3 \leq 0 - 3$

$x \leq -3$

Ex 2: Rational Function

$y = \frac{a}{b(x-h)} + k$

where $b(x-h) \neq 0$

ex find the domain

$f(x) = \frac{-2}{-x+1} + 3$

$-x + 1 - 1 \neq 0 - 1$ isolate x!

$\frac{\cancel{-x}}{\cancel{-1}} \neq \frac{-1}{-1}$

$x \neq 1$

Domain $\mathbb{R} \setminus \{1\}$

Substracting 2 Functions

QI: if $f(x) = 2$
$g(x) = x^2 + 2$

a) find $f-g(x)$
b) find $f-g(1)$
c) Does g and $f-g$ have the same range?

$$\boxed{f-g(x) = f(x) - g(x)}$$

$$= 2 - (x^2 + 2)$$

$$= 2 - x^2 - 2$$

ANS $\boxed{f-g(x) = -x^2}$

b) $f-g(1)$
$f-g(x) = -x^2$
$f-g(1) = -(1^2)$
$f-g(1) = -1$

b) $f-g(1) = f(1) - g(1)$

True!

$= 2 - 3$

$= -1$

if $f(x) = 2$

$y = a(x-h)^2 + k$

$g(x) = x^2 + 2$ $\quad V(0,2)$

$y = ax^2 + bx + c$

c) Does g and $f-g$ have the same range? No

Recall: graphing parabola

step ① vertex (h,k)

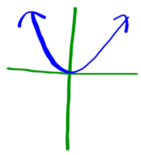
$v = (0,0)$

step ②

x	y
$h-2$	
$h-1$	
h	k
$h+1$	
$h+2$	

step ③ plot points.

ANS $\boxed{f - g(x) = -x^2}$

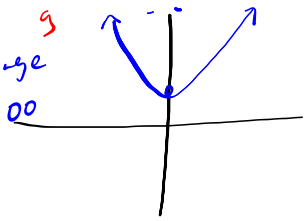


$y = a(x-h)^2 + k$

$h = 0 \quad k = 0$

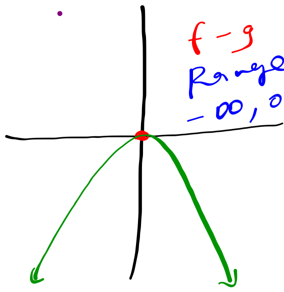
graph/sketch g

Range $\{2, \infty$



$f-g$

Range $-\infty, 0]$



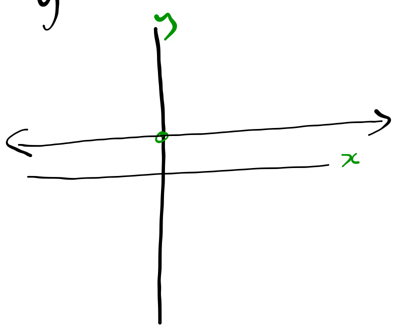
d) Does f and $f-g$ have same range?

Recall: horizontal line graph

$y = k$

i.e. $f(x) = 2$

$y = 2$



Multiplying 2 Functions

if $f(x) = \sqrt{x} + 2$
$g(x) = \sqrt{x} + 1$

a) find $f \cdot g(4)$
b) find domain of $f \cdot g$

Recall:
2. $r\sqrt[2]{a} \times s\sqrt[2]{b} = r \times s\sqrt[2]{a \times b}$

1. $\sqrt{x} \cdot \sqrt{x} = \sqrt{x \cdot x}$
$= \sqrt{\cancel{x}\cancel{x}}$
$= x$

$2 \cdot \sqrt{x} = 2\sqrt{x}$

$$\boxed{f \cdot g(x) = f(x) \cdot g(x)}$$

$f \cdot g(x) = (\sqrt{x} + 2) \cdot (\sqrt{x} + 1)$
$= \sqrt{x} \cdot \sqrt{x} + \sqrt{x} + 2\sqrt{x} + 2$
$= x + 1\sqrt{x} + 2\sqrt{x} + 2$

$f \cdot g(x) = x + 3\sqrt{x} + 2$

a) find $f \cdot g(4) = 4 + 3\sqrt{4} + 2$
$= 4 + 6 + 2$
$= 12$

Recall : graphing square root function

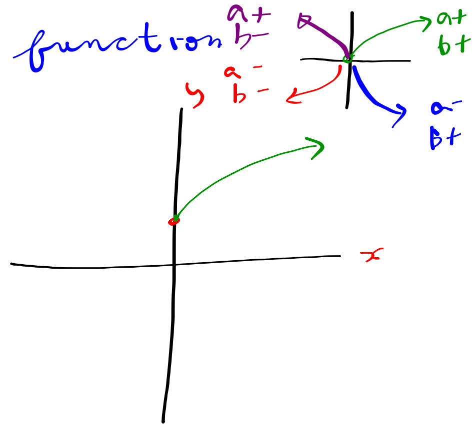
$f(x) = \sqrt{x} + 2$

$= a\sqrt{b(x-h)} + k$

step i find parameter & vertex

$a = 1$ $h = 0$
$b = 1$ $k = 2$ } $v(0, 2)$

step ii consider how a & b affects graph



p 1.8 # 4

$f(x) = \frac{1}{x+2}$

$g(x) = x^2$

find $f \cdot g(x)$

find $f \cdot g(4)$

find domain of $f \cdot g$

$f \cdot g(x) = f(x) \cdot g(x)$

$f \cdot g(x) = \frac{1}{x+2} \cdot \frac{x^2}{1}$

$\boxed{f \cdot g(x) = \frac{x^2}{x+2}}$

can't cancel out!!

$f \cdot g(4) = \frac{4^2}{4+2}$

$f \cdot g(4) = \frac{16}{6}$

$f \cdot g(4) = \frac{8}{3}$

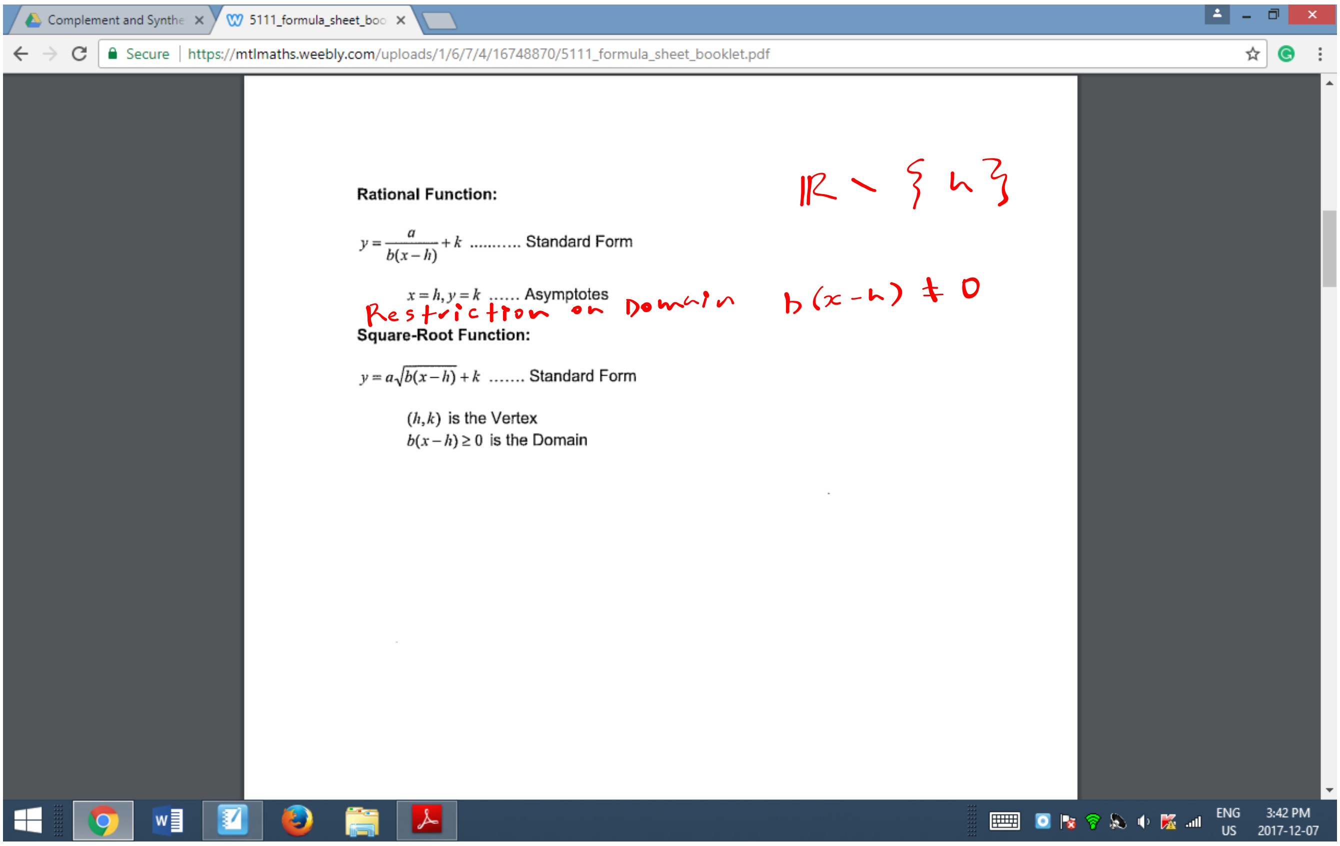
domain $f \cdot g$

$x + 2 \neq 0$

$x \neq -2$

$\mathbb{R} \setminus \{-2\}$ Dom

$]-\infty, -2[\cup]-2, \infty[$ Dom



Rational Function:

$\mathbb{R} \setminus \{h\}$

$y = \frac{a}{b(x-h)} + k$ .... Standard Form

$x = h, y = k$ ...... Asymptotes

Restriction on Domain $b(x-h) \neq 0$

Square-Root Function:

$y = a\sqrt{b(x-h)} + k$ .... Standard Form

(h,k) is the Vertex

$b(x-h) \geq 0$ is the Domain

Recall: graphing rational Function

$$y = \frac{-1}{x+2} - 3$$

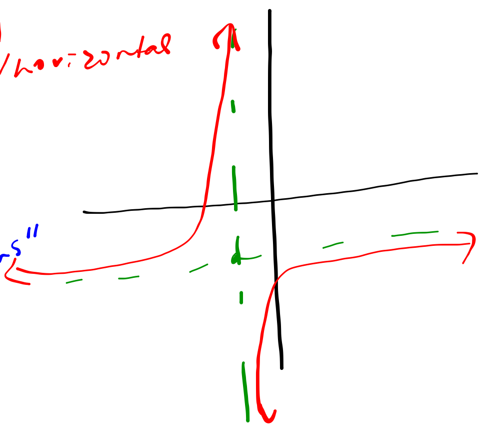
$$y = \frac{a}{b(x-h)} + k$$

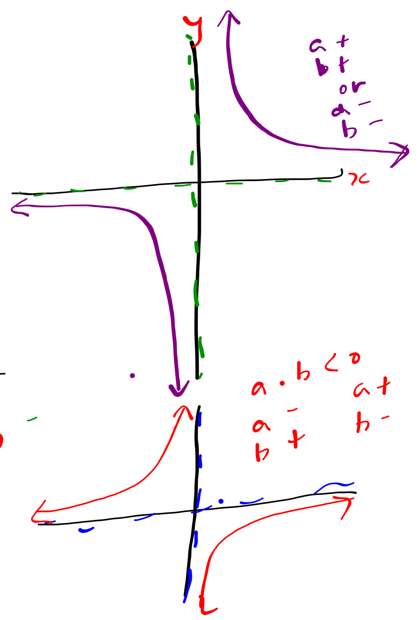
step i:

plot (h,k) and draw vertical/horizontal lines

step ii

consider a and b to figure out correct "quadraters"





$f(x) = x^2$

$g(x) = \frac{1}{x}$ $x \neq 0$

$h(x) = x^2$

$v(x) = \frac{1}{x+1}$

$h \cdot v = \frac{x^2}{x+1}$

$f \cdot g$

$f \cdot g(x) = \frac{x^2}{x}$

$f \cdot g(x) = x$ $x \neq 0$

Dividing 2 Functions

$$\boxed{f \div g(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0}$$

if $f(x) = 2x$
$g(x) = \sqrt{2x}$

find $f \div g(x)$

step i Do operation

step ii. Simplify by converting radicals to exponents and then using laws of exp. by factoring and canceling only when one term in top and bottom!

(law 2)

Step iii Find the domain by taking intersection of the function domains of each and the fact that $g(x) \neq 0$

$$f \div g(x) = \frac{2x}{\sqrt[2]{(2x)^1}}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$= \frac{(2x)^1}{(2x)^{\frac{1}{2}}}$$

$$= (2x)^{\frac{2}{2} - \frac{1}{2}}$$

$$= (2x)^{\frac{1}{2}}$$

$$= \sqrt{2x}$$

$$= \frac{2^1 x^1}{2^{\frac{1}{2}} x^{\frac{1}{2}}}$$

$$= 2^{1-\frac{1}{2}} x^{1-\frac{1}{2}}$$

$$= 2^{1/2} x^{1/2}$$

$$= (2x)^{1/2}$$

$$= \sqrt{2x}$$

ANS $\boxed{f \div g(x) = \sqrt{2x}}$

$2x \geq 0$
$x \geq 0$

$g(x) = \sqrt{2x}$
$g(x) \neq 0$
$(\sqrt{2x})^2 \neq 0^2$
$\frac{\cancel{2}x}{\cancel{2}} \neq \frac{0}{2}$
$x \neq 0$

Domain $x > 0$
$]0, \infty$

if $f(x) = x + 4$

$g(x) = x^2 + 6x + 8$

find $f \div g\,(x)$

find domain

$ax^2 + bx + c$

$\underline{a \cdot c}$

to find factors that add up to

8

8 1

2 4

-8 -1

-2 -4

$f \div g\,(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

$f \div g\,(x) = \dfrac{x+4}{x^2+6x+8}$

$= \dfrac{\cancel{(x+4)}}{(x+2)\cancel{(x+4)}}$

$$\boxed{f \div g(x) = \frac{1}{x+2}}$$

Restriction

$x + 2 \neq 0$

$x \neq -2$

$g(x) \neq 0$

$x^2 + 6x + 8 \neq 0$

$(x+2)(x+4) \neq 0$

$\neq 0$ $\neq 0$

and $x + 4 \neq 0$

$x \neq -4$

Dom: $\mathbb{R} \setminus \{-2, -4\}$

if $f(x) = x - 2$

$g(x) = x^2 - 4$

find $\frac{g}{f}$ $f(x) \neq 0$

find domain

$$\frac{g}{f}(x) = \frac{x^2 - 4}{x - 2}$$

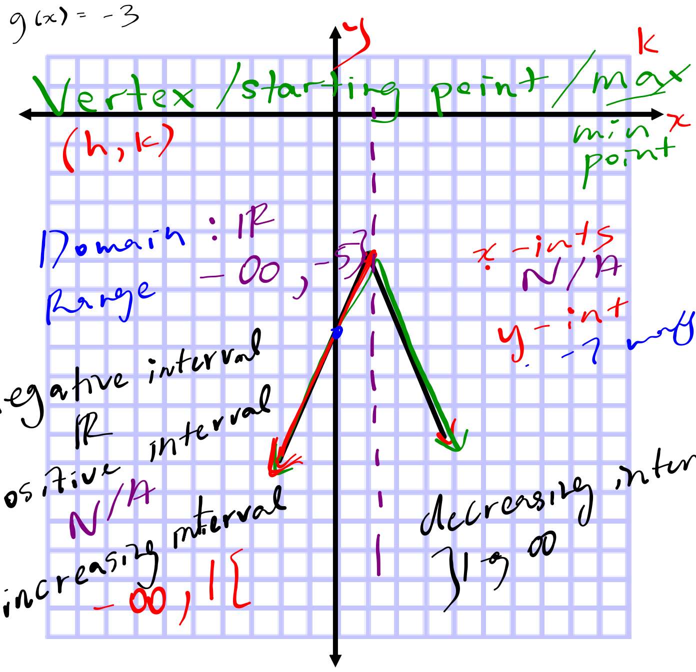
$$= \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$g/ = x + 2$$

Unit 2 : Characteristics of the function resultating from an operation $(+, -, \times, \div)$

$f(x) = 2|x-1| + 2$

$g(x) = -3$



Vertex / starting point / max
min point

(h, k)

Domain : $\mathbb{R}$

Range $-\infty, -5]$

x-ints N/A

y-int -7

negative interval $\mathbb{R}$

positive interval N/A

increasing interval $-\infty, 1[$

decreasing interval $]1, \infty$

$g - f(x) = g(x) - f(x)$

$g - f(x) = -3 - (2|x-1| + 2)$

$g - f(x) = \underline{-3} - 2|x-1| \underline{-2}$

$g - f(x) = -2|x-1| - 5$

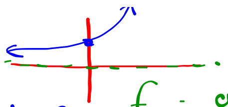
$y = a|x-h| + k$

step 1 find (h, k)

x	y
h	k

asymptote / axis of symmetry

if $f(x) = a\,c^{b(x-h)} + k$



c does b's job

$f \cdot g$

$f(x) = 2^x + 3$

$g(x) = -3$

HMWK

p 1.8

p 1.13 – 1.18 not every question

p 2.29 (p 2.11)

don't do step f.[x]

Q I Are both $f \cdot g$ and f increasing over their entire domain?

Q II Is the range of $f \cdot g$ a subset of f

Q III True/False

$f \cdot g$ is negative over its whole domain and f is positive over its whole domain

* Partial True is False!