Word Questions for Log and Exp Function

Q1. Your educational savings plan has an annual interest rate of 2% compounded monthly. If you initially put in \$ 15 000, how long would you have to wait to see a return of \$ 40 000?

$t = ?$ $A(t)$

$$A(t) = A_0\left(1+\frac{r}{k}\right)^{k\cdot t}$$

$A_0 = 15\,000$

$r = 0.02$

$t \rightarrow$ in years
(since annual interest rate)

$k = 12$

$$A(t) = 15000\left(1+\frac{0.02}{12}\right)^{12\cdot t}$$

$$\frac{40\,000}{15\,000} = \frac{\cancel{15000}\left(1+\frac{0.02}{12}\right)^{12\cdot t}}{\cancel{15000}}$$

$$\frac{40000}{15000} = \left(1+\frac{0.02}{12}\right)^{12\cdot t}$$

$$\frac{\log_{\left(1+\frac{0.02}{12}\right)}\frac{40000}{15000}}{12} = \frac{\cancel{12}\cdot t}{\cancel{12}}$$

$$t = \frac{\frac{\log \frac{40\,000}{15\,000}}{\log\left(1+\frac{0.02}{12}\right)}}{12} \qquad t = 49$$

It will take 49 years to make an investment of \$ 40 000.

Q2. The swine flu bacteria triple every 20 mins. If there were initially 5 bacterium in the classroom, how long would it take to have 100 bacterium?

$t = ?$ $A(t)$

$$A(t) = ac^{kt}$$

$a = 5$

$c = 3$

time $\rightarrow$ hours

$k = \frac{\text{unit of time}}{\text{given time}}$

$k = \frac{60 \text{ mins}}{20 \text{ mins}}$

$k = 3$

$$A(t) = 5\cdot 3^{3\cdot t}$$

$$\frac{100}{5} = \frac{\cancel{5}\cdot 3^{3\cdot t}}{\cancel{5}}$$

$$20 = 3^{3t}$$

$$\frac{\log_3 20}{3} = \frac{\cancel{3}t}{\cancel{3}}$$

$$t = \frac{\frac{\log 20}{\log 3}}{3}$$

$$t = 0.909$$

It takes 0.909 hours for there to be 100 bacterium.

Q3. A chain email gets automatically sent to 10 new people every half hour. If 5 people initially sent out the email, how long would you have to wait until 10 000 people see the email?

$t = ?$ $A(t)$

$A(t) = a c^{k \cdot t}$

$a = 5$

$c = 10$

$t \rightarrow$ in hours

$k = \frac{\text{unit of time}}{\text{given time}}$

$k = \frac{60 \text{ mins}}{30 \text{ mins}}$

$k = 2$

$A(t) = 5 \cdot 10^{2 \cdot t}$

$\frac{10000}{5} = \frac{\cancel{5} \cdot 10^{2 \cdot t}}{\cancel{5}}$

$2000 = 10^{2 \cdot t}$

$\frac{\log_{10} 2000}{2} = \frac{\cancel{2} t}{\cancel{2}}$

$t = \frac{\frac{\log 2000}{\log 10}}{2}$

$t = 1.65$

It takes 1.65 hours.

Q4. A rumour gets told to three new people every two hours. If 8 people know the rumour at first, how many people would know the rumour after 8 hours?

$A(t) = ?$ t find $A(8)$

$A(t) = a c^{k \cdot t}$

$a = 8$

$c = 3$

$t \rightarrow$ hours

$k = \frac{\text{unit of time}}{\text{given time}}$

$k = \frac{1 \text{ hr}}{2 \text{ hr}}$ $k = \frac{1}{2}$

$A(t) = 8 \cdot 3^{1/2 \cdot t}$

$A(8) = 8 \cdot 3^{1/2(8)}$

$A(8) = 648$

648 people know the rumour after 8 hours

Q5. A bacteria populations becomes 2.2 times larger every three hours. If there's 3 bacterium at first, what's the population after 24 hours?

$A(t) = ?$ $t = 24$

$A(t) = a \cdot c^{kt}$

$a = 3$

$c = 2.2$

$t \rightarrow$ hours

$k = \frac{\text{unit of time}}{\text{given time}}$

$k = \frac{1 \text{ hr}}{3 \text{ hr}}$ $k = \frac{1}{3}$

$A(t) = 3 \cdot 2.2^{1/3 t}$

$A(24) = 3 \times 2.2^{1/3(24)}$

$A(24) = 1646.28$

There will be 1646.28 bacterium after 24 hours.